CHAPTER 6

Lines and Angles

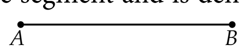
Introduction

In previous class we have studied that minimum two points are required to draw a line. A line having one end point is called a ray. Now if two rays originate from a point, an angle is formed. If two lines intersect each other, different angles are formed. Here we will study different properties related to lines and angles.

Basic Terms and Definitions

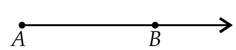
Line Segment

A part of a line with two end points is called line segment and is denoted as $\overline{AB}$.



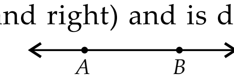
Ray

A part of a line with one end point is called ray and is denoted as $\overrightarrow{AB}$. It can be extended further from point *B*.



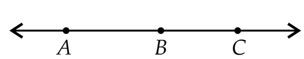
Line

It can be extended from both sides (left and right) and is denoted as $\overleftrightarrow{AB}$.



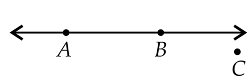
Collinear Points

Three or more points are said to be collinear if a single straight line passes through them. Here *A*, *B*, *C* are collinear.



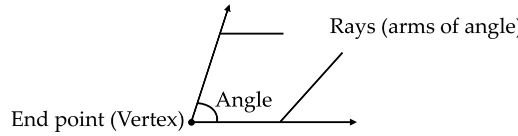
Non-Collinear Points

Three or more points not lying on a single straight line are called non-collinear points. *A*, *B*, *C* are not collinear.



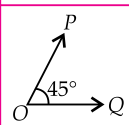
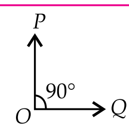
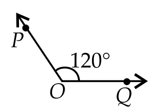
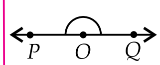
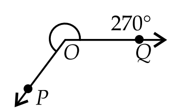
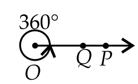
Angle

When two rays originates from the same end point they form an angle.

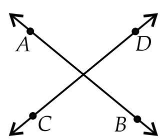


The rays are called *arms of an angle* and end point is called *vertex*.

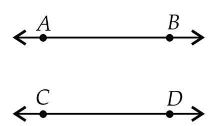
Types of Angles

Measure	$0 < x < 90°$	$x = 90°$	$90° < x < 180°$	$180°$	$180° < x < 360°$	$360°$
Name	Acute angle	Right angle	Obtuse angle	Straight angle	Reflex angle	Complete angle
Illustration						

Intersecting Lines and Non-Intersecting Lines



(i) Intersecting lines



(ii) Non- intersecting lines (Parallel Lines)

In figure (i) AB and CD are intersecting lines.

In figure (ii) AB and CD are non-intersecting lines (parallel lines).

In parallel lines the lengths of the common perpendiculars at different points are equal. This equal length is called the distance between two parallel lines.

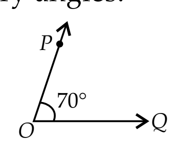
Pairs of Angles

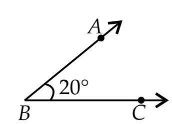
• Complementary Angles

If the sum of measure of two angles is 90°, they are known as complementary angles.

For example : $\angle POQ + \angle ABC = 70° + 20° = 90°$

$\therefore$ $\angle POQ$ and $\angle ABC$ are complementary angles and $\angle POQ$ is called complement of $\angle ABC$ and vice-versa.



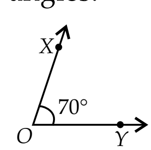


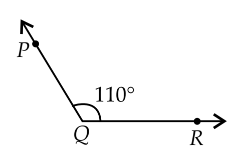
• Supplementary Angles

If the sum of measure of two angles is 180°, they are called supplementary angles.

For example : $\angle XOY$ and $\angle PQR$ are supplementary as

$\angle XOY + \angle PQR = 70° + 110° = 180°$ and $\angle XOY$ is called supplement of $\angle PQR$ and vice-versa.





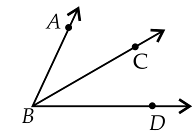
• Adjacent Angles

Two angles are said to be adjacent angles or adjacent to each other if

(i) They have common arm.

(ii) They have common vertex.

(iii) Non-common arms lying on the different sides of the common arm.

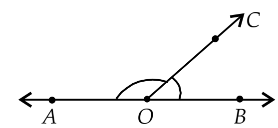


In the figure, $\angle ABC$ and $\angle CBD$ are adjacent angles.

They have common vertex B, common arm BC and non-common arms AB and BD lying on the different sides of BC.

• Linear Pair

Two adjacent angles whose sum is 180° are said to form linear pair or in other words, supplementary adjacent angles are called linear pair.

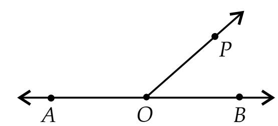


Here, $\angle BOC + \angle COA = 180°$, so they form linear pair.

Linear Pair Axiom

Axiom-1

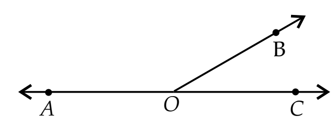
If a ray stands on a line, then the sum of two adjacent angles so formed is 180°. Ray OP stands on line AB, then $\angle AOP$ and $\angle POB$ are adjacent angles.



$\Rightarrow \quad \angle AOP + \angle POB = 180°$

Axiom-2

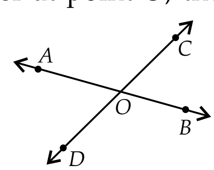
If the sum of two adjacent angles is 180°, then the non-common arms of the angles form a straight line. Suppose $\angle AOB$ and $\angle BOC$ are two adjacent angles, with common arm OB, non common arms OA and OC.



If $\angle AOB + \angle BOC = 180°$, then AOC would be a straight line.

Vertically Opposite Angles

If two lines AB and CD intersects each other at point O, then four angles are formed.



$\angle AOD$ is vertically opposite to $\angle BOC$. Similarly $\angle AOC$ is vertically opposite to $\angle DOB$. These are called pairs of vertically opposite angles.

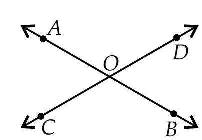
Theorem 1

Statement : If two lines intersect each other, then the vertically opposite angles are equal.

Given : AB and CD are two lines intersecting each other at point O. $\angle AOC$ is vertically opposite to $\angle DOB$ and $\angle AOD$ is vertically opposite to $\angle COB$.

To Prove : $\angle AOC = \angle DOB$

$\angle AOD = \angle COB$



Proof : $\angle AOC + \angle AOD = 180°$ ...(1) [Linear pair]

$\angle AOD + \angle DOB = 180°$ ...(2) [Linear pair]

From (1) and (2), we get

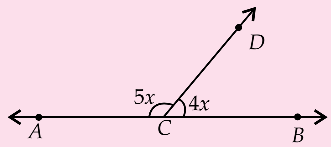
$\angle AOC + \angle AOD = \angle AOD + \angle DOB$

$\Rightarrow \quad \angle AOC = \angle DOB$

Similarly, $\quad \angle AOD = \angle COB$

ILLUSTRATION

1 ACB is a line such that $\angle DCA = 5x$ and $\angle DCB = 4x$. Find the value of x and hence find $\angle DCA$ and $\angle DCB$.



Soln.: Since ACB is a line,

$\therefore$ $\angle ACD$ and $\angle DCB$ form a linear pair

$\Rightarrow$ $\angle ACD + \angle DCB = 180° \Rightarrow 5x + 4x = 180°$

$9x = 180° \Rightarrow x = 20°$

$\therefore$ $\angle ACD = 5x = 5 \times 20° = 100°$

$\angle DCB = 4x = 4 \times 20° = 80°$

2 If the supplement of an angle is two-third of itself. Determine the angle and its supplement.

Soln.: Let the angle be x

$\therefore$ Its supplement $= \frac{2}{3}$ of $x = \frac{2}{3}x$

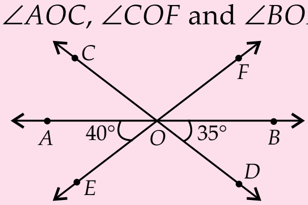
$\Rightarrow x + \frac{2}{3}x = 180° \Rightarrow \frac{3x + 2x}{3} = 180°$

$\Rightarrow \frac{5x}{3} = 180° \Rightarrow x = \frac{180° \times 3}{5} = 108°$

$\therefore$ The angle is 108°

and its supplement $= \frac{2}{3} \times 108° = 2 \times 36° = 72°$

3 Lines AB, CD and EF intersect at O. Find the measures of $\angle AOC$, $\angle COF$ and $\angle BOF$



Soln.: AB and EF intersect at point O.

$\therefore$ $\angle AOE = \angle FOB$ [Vertically opposite angles]

$\therefore$ $\angle FOB = 40°$

Similarly, $\angle AOC = \angle DOB$

[Vertically opposite angles]

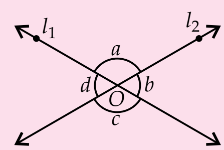
$\Rightarrow$ $\angle AOC = 35°$

Also, $\angle AOE + \angle AOC + \angle COF = 180°$

[Straight angle]

$\angle COF = 180° – 75° = 105°$

4 Lines l_1 and l_2 intersects at point O forming angles a, b, c, d.



If $a = 45°$, find b, c, d, $a + b$, $b + c$ and verify that $a + d = b + c$.

Soln.: $a = 45° \Rightarrow c = 45°$

[Vertically opposite angles]

$a + d = 180°$ [Linear pair]

$\Rightarrow$ $d = 180° – a = 180° – 45° = 135°$

$d = b$ [Vertically opposite angles]

$\Rightarrow$ $b = 135°$

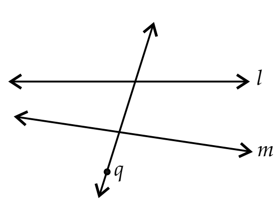
Now, $a + d = 45° + 135° = 180°$

$b + c = 135° + 45° = 180°$

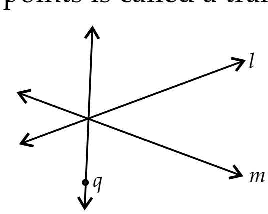
$\Rightarrow$ $a + d = b + c$

Transversal

A line which intersects two or more lines at distinct points is called a transversal.



(q is a transversal of l and m)



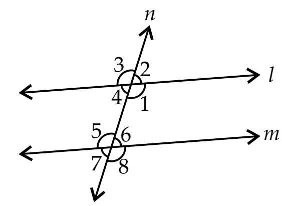
(q is not a transversal of l and m)

Angles Formed by a Transversal

Suppose l and m are two parallel lines intersected by a transversal n.

Interior Angles

$\angle 1, \angle 4, \angle 5, \angle 6$



Exterior Angles

$\angle 3, \angle 2, \angle 7, \angle 8$

Corresponding Angles : $\angle 1$ and $\angle 8$, $\angle 2$ and $\angle 6$, $\angle 3$ and $\angle 5$, $\angle 4$ and $\angle 7$

[Note that pairs of angles lie either above or below the lines.]

Alternate Interior Angles : $\angle 4$ and $\angle 6$, $\angle 1$ and $\angle 5$

Alternate Exterior Angles : $\angle 3$ and $\angle 8$, $\angle 2$ and $\angle 7$

Co-Interior Angles : (Also called consecutive interior angles or allied angles) $\angle 4$ and $\angle 5$, $\angle 1$ and $\angle 6$.

Co-Exterior Angles : $\angle 2$ and $\angle 8$, $\angle 3$ and $\angle 7$.

Results when a Transveral Intersects Two Parallel lines

Corresponding Angles Axiom

(i) If a transversal intersects two parallel lines, then each pair of corresponding angles are equal.

(ii) If a transversal intersects two lines such that a pair of corresponding angles are equal, then the two lines are parallel to each other.

Alternate Interior Angles Theorem

(i) If a transversal intersects two parallel lines, then each pair of alternate interior angles are equal.

(ii) If a transversal intersects two lines such that a pair of alternate interior angles are equal, then the two lines are parallel.

Co-interior Angles Theorem

(i) If a transversal intersects two parallel lines, then each pair of interior angles on the same side of the transversal is supplementary. In other words, co-interior angles are supplementary.

(ii) If a transversal intersects two lines such that a pair of co-interior angles are supplementary, then the two lines are parallel.

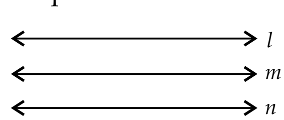
Co-exterior Angles Theorem

(i) If a transversal intersects two parallel lines then each pair of co-exterior angles are supplementary.

(ii) If a transversal intersects two lines such that a pair of co-exterior angles are supplementary, then two lines are parallel.

Parallel Lines Theorem

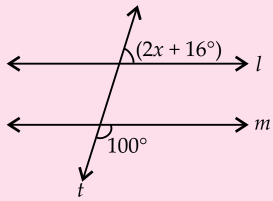
Lines which are parallel to the same line are parallel to each other.



This means if $l \parallel n$ and $m \parallel n \Rightarrow l \parallel m$.

ILLUSTRATION

5 In the given figure if $l \parallel m$ and t is a transversal, determine x.

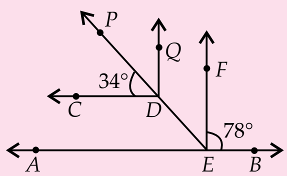


Soln.: $l \parallel m$

$\Rightarrow \quad 2x + 16° + 100° = 180°$ [Co-exterior angles]

$\Rightarrow \quad 2x = 180° - 116° \Rightarrow 2x = 64° \Rightarrow x = 32°$

6 $AB \parallel CD$ and $EF \parallel DQ$. Determine $\angle PDQ$, $\angle AED$ and $\angle DEF$.



Soln.: $AB \parallel CD$ and transveral DE intersects them at E and D respectively.

$\therefore \quad \angle AED = \angle CDP$ [Corresponding angles]

$\Rightarrow \quad \angle AED = 34°$ ... (i)

Now ray EF stands on AB at E

$\therefore \quad \angle AEF + \angle BEF = 180°$ [Linear pair]

$\Rightarrow \quad \angle AEP + \angle PEF + \angle BEF = 180°$

[$\angle AEF = \angle AEP + \angle PEF$]

$\Rightarrow \quad 34° + \angle PEF + 78° = 180°$ [From (i)]

$\Rightarrow \quad \angle PEF = 180° - 112°$

$\Rightarrow \quad \angle PEF = 68°$ ...(ii)

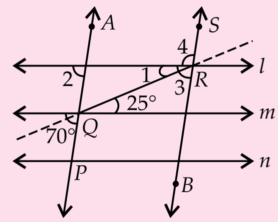
Now $EF \parallel DQ$ and transversal DE intersects them at E and D respectively.

$\therefore \quad \angle FED = \angle PDQ$ [Corresponding angles]

$\Rightarrow \quad \angle PDQ = 68°$ [From (ii)]

So, $\angle PDQ = \angle DEF = 68°$ and $\angle AED = 34°$

7 If $l \parallel m \parallel n$ and $PQ \parallel RS$, find $\angle QRS$.



Soln.: Extend QP to point A and SR to point B.

$l \parallel m \Rightarrow \angle 1 = 25°$ [Alternate interior angles]

and $\angle 2 = 70°$ [Corresponding angles]

Also, $\angle 3 = \angle 2$ [$AP \parallel SB$, corresponding angles]

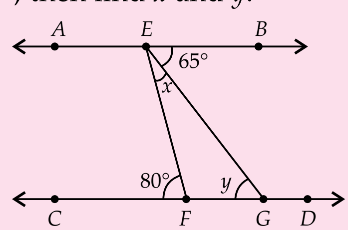
$\Rightarrow \quad \angle 3 = 70°$

Now, $\angle 3 + \angle 4 = 180°$ [Linear pair]

$\Rightarrow \quad 70° + \angle 4 = 180° \Rightarrow \angle 4 = 110°$

$\therefore \quad \angle QRS = \angle 1 + \angle 4 = 25° + 110° = 135°$

8 In figure, if $AB \parallel CD$, $\angle BEG = 65°$ and $\angle EFC = 80°$, then find x and y.



Soln.: $\angle BEF = \angle EFC$ [Alternate interior angles]

$\Rightarrow \quad 65° + x = 80°$

$\Rightarrow \quad x = 80° - 65° \Rightarrow x = 15°$

Now, $\angle FGE = \angle BEG$ [Alternate interior angles]

$\Rightarrow \quad y = 65°$.

Properties of Triangles

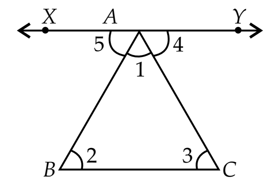
- ### Angle Sum Property of a Triangle

Theorem 2

Statement : The sum of the three angles of a triangle is 180°.

i.e., $\angle 1 + \angle 2 + \angle 3 = 180°$

Proof : Draw a line XY through point A parallel to BC.



$\angle 3 = \angle 4$ [Alternate interior angles]

$\angle 5 = \angle 2$ [Alternate interior angles]

Also $\angle 5 + \angle 1 + \angle 4 = 180°$ [Straight angle]

Replacing $\angle 5$ and $\angle 4$ by $\angle 2$ and $\angle 3$ respectively, we get

$\angle 2 + \angle 1 + \angle 3 = 180°$

$\Rightarrow$ $\angle 1 + \angle 2 + \angle 3 = 180°$

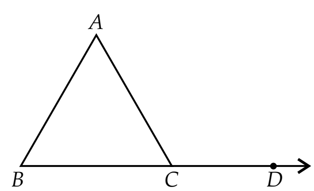
$\therefore$ Sum of all angles of a triangle is 180°.

• Exterior Angle Property of a Triangle

Theorem 3

Statement : If a side of a triangle is produced, then the exterior angle so formed is equal to the sum of the two interior opposite angles.

Given : In ΔABC, side BC is extended to point D. $\angle ACD$ is an exterior angle.



To Prove : $\angle ACD = \angle CAB + \angle ABC$

Proof : $\angle ABC + \angle CAB + \angle BCA = 180°$ ...(1) [Angle sum property of triangle]

Also, $\angle BCA + \angle ACD = 180°$ ...(2) [Linear pair]

From (1) and (2), we get

$\Rightarrow$ $\angle ABC + \angle CAB + \angle BCA = \angle BCA + \angle ACD$

$\Rightarrow$ $\angle ACD = \angle ABC + \angle CAB$

ILLUSTRATION

9 If one angle of a triangle is 72° and the difference of the other two angles is 12°, find the other two angles.

Soln.: One angle of the triangle = 72°

Let other two angles be x and $12° + x$

[$\because$ The difference between the two angles is 12°]

So, $x + 12° + x + 72° = 180°$

[Angle sum property of triangle]

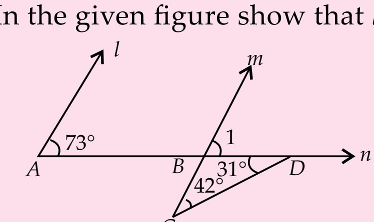
$\Rightarrow$ $2x + 84° = 180°$

$\Rightarrow$ $2x = 180° - 84° = 96°$

$\Rightarrow$ $x = \dfrac{96°}{2} = 48°$

$\therefore$ The other angles are 48° and 48° + 12° = 60°

10 In the given figure show that $l \parallel m$



Soln.: $\angle CBD + \angle BDC + \angle BCD = 180°$

[Angle sum property of triangle]

$\Rightarrow$ $\angle CBD + 31° + 42° = 180°$

$\Rightarrow$ $\angle CBD = 180° - 73°$

$\Rightarrow$ $\angle CBD = 107°$

ILLUSTRATION

$\angle 1 + \angle CBD = 180°$ [Linear pair]

$\angle 1 = 180° - 107° = 73°$

Since, $\angle 1 = 73°$ and $\angle A = 73°$

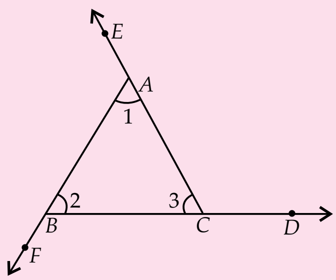
So, $\angle 1 = \angle A$

$\Rightarrow$ $l \parallel m$ [As corresponding angles are equal]

11 The sides BC, CA and AB of ΔABC, are produced in order, forming exterior angles $\angle ACD$, $\angle BAE$ and $\angle CBF$. Prove that

$\angle ACD + \angle BAE + \angle CBF = 360°$.

Soln.:



From exterior angle property of triangle,

$\angle FBC = \angle 1 + \angle 3$ ...(i)

$\angle BAE = \angle 2 + \angle 3$ ...(ii)

$\angle ACD = \angle 1 + \angle 2$ ...(iii)

Adding (i), (ii) and (iii), we get

$\angle FBC + \angle BAE + \angle ACD$

$= \angle 1 + \angle 3 + \angle 2 + \angle 3 + \angle 1 + \angle 2$

$= 2(\angle 1 + \angle 2 + \angle 3) = 2 \times 180°$

[By angle sum property of triangle]

$\Rightarrow$ $\angle FBC + \angle BAE + \angle ACD = 360°$

ESSENTIAL POINTS for COMPETITIVE EXAMS

Angle

An angle is formed when two rays originate from the same point.

Some Angle Relations

- **Adjacent Angles**

  Two angles are called adjacent angles, if

  (i) they have same vertex

  (ii) they have common arm

  (iii) uncommon arms are on different sides of the common arm

- **Linear Pair of Anlges**

  Two adjacent angles are said to form a linear pair of angles, if

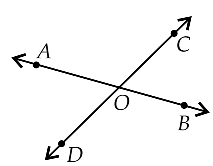
  (i) their non-common arms are two opposite rays

  (ii) the sum of the adjacent angles so formed is 180°

  (iii) they are supplementary.

- **Vertically Opposite Angles**

  If two lines AB and CD intersects each other at point O, then $\angle AOD$ is vertically opposite to $\angle BOC$. Similarly $\angle AOC$ is vertically opposite to $\angle DOB$.



- **Transversal**

  A line which intersects two or more given lines at distinct points, is called a transversal of the given lines.

- The measures of vertical opposite angles are equal.
- The measures of angles forming a linear pair has total 180°.
- **Angle Relationships Formed by Parallel Lines being cut by a Transversal .**

  (i) The measures of alternate exterior angles are equal.

  (ii) The measures of alternate interior angles are equal.

  (iii) The measures of the same-side interior angles are supplementary (sum to 180°)

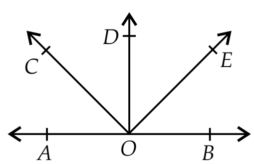
  (iv) The measures of corresponding angles are equal

- Lines which are parallel to a given line are parallel to each other.
- Sum of three angles of a triangle is 180°.
- If a side of a triangle is produced, the exterior angle so formed is equal to the sum of the two interior opposite angles.

SOLVED EXAMPLES

1. Prove that the sum of all angles formed on the same side of a line at a given point on the line is 180°.

Soln.: Given : AOB is a straight line and rays OC, OD and OE stands on it, forming $\angle AOC$, $\angle COD$, $\angle DOE$ and $\angle EOB$.



To Prove :

$\angle AOC + \angle COD + \angle DOE + \angle EOB = 180°$.

Proof : Ray OC stands on line AB.

$\therefore \angle AOC + \angle COB = 180°$ [Linear pair]

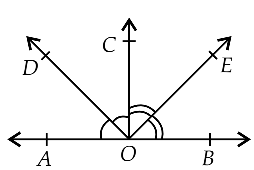
$\Rightarrow \angle AOC + (\angle COD + \angle DOE + \angle EOB) = 180°$

$\Rightarrow \angle AOC + \angle COD + \angle DOE + \angle EOB = 180°$.

Hence, the sum of all the angles formed on the same side of line AB at a point O on it is 180°.

2. Prove that the bisectors of the angles of a linear pair are at right angle.

Soln.: Given : $\angle AOC$ and $\angle BOC$ form a linear pair of angles. OD and OE are the bisectors of $\angle AOC$ and $\angle BOC$ respectively.



To Prove : $\angle DOE = 90°$.

Proof : $\angle AOC + \angle BOC = 180°$ [Linear pair]

$$\Rightarrow \frac{1}{2}\angle AOC + \frac{1}{2}\angle BOC = 90°$$

$\Rightarrow \angle DOC + \angle COE = 90°$

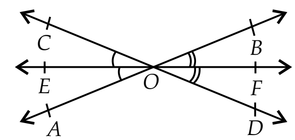
[$\because$ OD and OE are the bisectors of $\angle AOC$ and $\angle BOC$]

$\Rightarrow \angle DOE = 90°$.

Hence, the bisectors of the angles of a linear pair are at right angle.

3. Prove that the bisectors of a pair of vertically opposite angles are in the same straight line.

Soln.: Given : Two lines AB and CD intersecting each other at a point O. Also, OE and OF are the bisectors of $\angle AOC$ and $\angle BOD$ respectively.



To Prove : EOF is a straight line.

Proof : Since, the sum of all angles around a point is 360°,

$\therefore \angle AOC + \angle BOC + \angle BOD + \angle AOD = 360°$

$\Rightarrow 2\angle EOC + 2\angle BOC + 2\angle BOF = 360°$

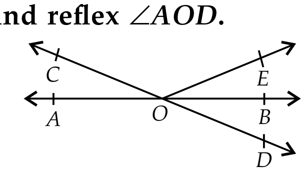
[$\because \angle BOC = \angle AOD$ (Vertically opposite angles). OE is bisector of $\angle AOC$, OF is bisector $\angle BOD$]

$\Rightarrow \angle EOC + \angle BOC + \angle BOF = 180°$

$\Rightarrow \angle EOF = 180°$

Hence, EOF is a straight line.

4. In the figure, lines AB and CD intersect at O. If $\angle AOC + \angle BOE = 80°$ and $\angle BOD = 30°$, find $\angle BOE$ and reflex $\angle AOD$.



Soln.: $\angle AOC + \angle BOE = 80°$ ...(i) [Given]

$\angle BOD = 30°$ ...(ii) [Given]

Lines AB and CD intersect at O.

$\therefore \angle AOC = \angle BOD$

[Vertically opposite angles]

$\Rightarrow \angle AOC = 30°$ [From (ii)]

Now, putting the value of $\angle AOC$ in (i), we have

$30° + \angle BOE = 80°$

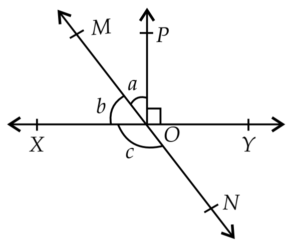
$\Rightarrow \angle BOE = 80° - 30° = 50°$.

Also, $\angle BOD + \angle AOD = 180°$ [Linear pair]

$\angle AOD = 180° - 30° = 150°$

and reflex $\angle AOD = 360° - 150° = 210°$.

5. In the figure, lines XY and MN intersect at O. If $\angle POY = 90°$ and $a = b$, find c.



Soln.: Given $a = b$

Let $a = x$ then $b = x$

$\angle XOM + \angle POM + \angle POY = 180°$ [Linear pair]

$\Rightarrow \quad x + x + 90° = 180°$

$\Rightarrow \quad 2x + 90° = 180° \quad \Rightarrow \quad 2x = 90°$

$\Rightarrow \quad x = \dfrac{90°}{2} = 45°$

$\therefore \quad \angle XOM = b = 45°$

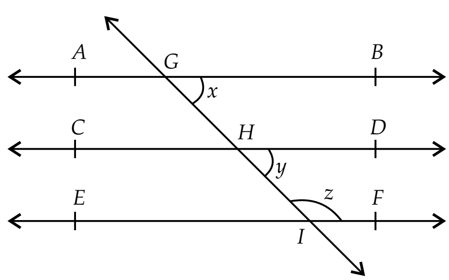
and $\quad \angle POM = a = 45°$

Now, $\quad \angle XON = c = \angle MOY$ [Vertically opposite angles]

$= \angle POM + \angle POY = 45° + 90°$

Hence, $c = 135°$.

6. In the figure, if $AB \parallel CD$, $CD \parallel EF$ and $y : z = 2 : 3$, find x.



Soln.: Let $y = 2a$ and $z = 3a$

$\angle DHI + \angle FIH = 180°$ [$CD \parallel EF$, Co-interior angles]

$\Rightarrow \quad y + z = 180°$

$\Rightarrow \quad 2a + 3a = 180°$

$\Rightarrow \quad 5a = 180°$

$\Rightarrow \quad a = \dfrac{180°}{5} = 36°$

$\therefore \quad y = 2a = 2 \times 36° = 72°$ and

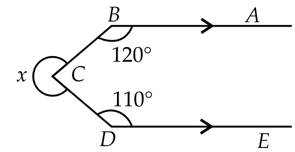
$z = 3a = 3 \times 36° = 108°$

Also, $AB \parallel CD$ and GI is a transversal

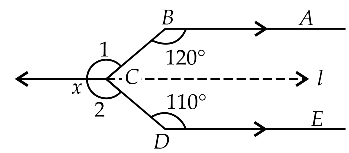
$\therefore \angle BGI = \angle DHI$ [Corresponding angles]

$\Rightarrow \quad x = y \quad \Rightarrow \quad x = 72°$

7. If $AB \parallel DE$, then find the value of x.



Soln.: Construct a line l through C parallel to AB.



$\Rightarrow \quad l \parallel AB \parallel DE$ and $x = \angle 1 + \angle 2$

Since, $AB \parallel l$

$\Rightarrow \quad \angle 1 = 120°$ [Alternate interior angles]

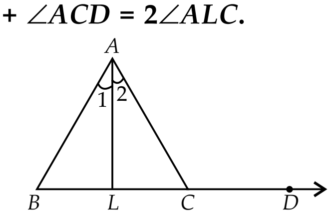
Also, $\quad l \parallel DE$

$\Rightarrow \quad \angle 2 = 110°$ [Alternate interior angles]

Now, $x = \angle 1 + \angle 2$

$\Rightarrow \quad x = 120° + 110° = 230°$

8. The side BC of a ΔABC is produced such that D is on ray BC. The bisector of $\angle A$ meets BC in L as in figure. Prove that $\angle ABC + \angle ACD = 2\angle ALC$.



Soln.: In ΔABC, we have

$\angle ACD = \angle B + \angle A$

[Exterior angle property of a triangle]

$\Rightarrow \quad \angle ACD = \angle B + 2\angle 1$ ...(i)

[$\because$ AL is the bisector of $\angle A$ $\therefore$ $\angle A = 2\angle 1$]

In ΔABL, we have

$\angle ALC = \angle B + \angle BAL$

[Exterior anlge property of a triangle]

$\Rightarrow \quad \angle ALC = \angle B + \angle 1$

$\Rightarrow \quad 2\angle ALC = 2\angle B + 2\angle 1$ ...(ii)

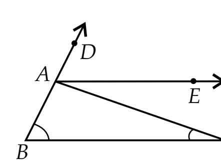
Subtracting (i) from (ii), we get

$2\angle ALC - \angle ACD = \angle B$

$\Rightarrow \quad \angle ACD + \angle B = 2\angle ALC$

$\Rightarrow \quad \angle ACD + \angle ABC = 2\angle ALC.$

9. AE bisects $\angle CAD$ and $\angle B = \angle C$, prove that $AE \parallel BC$.



Soln.: In ΔABC, we have

$\angle CAD = \angle B + \angle C$

[Exterior angle property of a triangle]

$\Rightarrow \angle CAD = 2\angle C$ [Given $\angle B = \angle C$]

$\Rightarrow 2\angle CAE = 2\angle C$ [$\because$ $\angle CAD$ is bisected by AE]

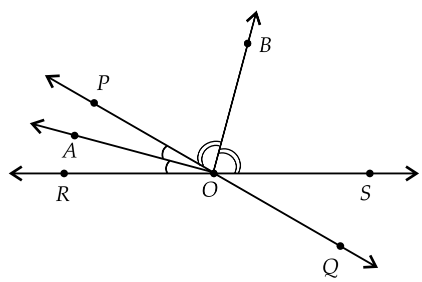
$\Rightarrow \angle CAE = \angle C$

$\Rightarrow \angle CAE = \angle ACB$

$\Rightarrow AE \parallel BC$

[As alternate interior angles are equal]

10. In figure, lines *PQ* and *RS* intersect each other at point *O*, ray *OA* and ray *OB* bisect $\angle POR$ and $\angle POS$ respectively. If $\angle POA : \angle POB = 2 : 7$, then find $\angle SOQ$ and $\angle BOQ$.



Soln.: $\angle POR + \angle POS = 180°$ [Linear pair]

We are given that, ray OA and ray OB bisect $\angle POR$ and $\angle POS$ respectively.

Therefore,

$$\angle POA = \frac{1}{2}\angle POR \text{ and } \angle POB = \frac{1}{2}\angle POS.$$

$$\Rightarrow \angle POA + \angle POB = \frac{1}{2}(\angle POR + \angle POS)$$

$$= \frac{1}{2} \times 180° = 90°$$

Now, if $\angle POA : \angle POB = 2 : 7$, then, we have

$$\angle POA = \frac{2}{9} \times 90° = 20° \text{ and}$$

$$\angle POB = \frac{7}{9} \times 90° = 70°.$$

$\angle POR = 2 \times \angle POA = 2 \times 20° = 40°$

$\angle SOQ = \angle POR$ [Vertically opposite angles]

$\therefore \angle SOQ = 40°$

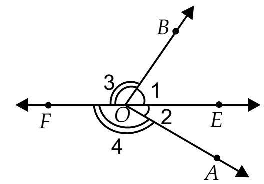
$\angle BOQ = \angle BOS + \angle SOQ = \angle POB + \angle SOQ$

$$\left[\angle BOS = \angle POB = \frac{1}{2}\angle POS\right]$$

$= 70° + 40° = 110°$

$\therefore \angle BOQ = 110°$.

11. Ray *OE* bisects $\angle AOB$ and *OF* is the ray opposite *OE*. Show that $\angle 3 = \angle 4$



Soln.: Ray OE and OF are opposite. So, FOF is a straight line.

$\therefore \angle 3 + \angle 1 = 180°$ (Linear pair) ...(i)

and $\angle 4 + \angle 2 = 180°$ (Linear pair) ...(ii)

From (i) and (ii), we have

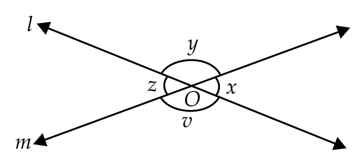
$\angle 3 + \angle 1 = \angle 4 + \angle 2$

$\Rightarrow \angle 3 + \angle 1 = \angle 4 + \angle 1$

($\because$ OE bisect $\angle AOB \Rightarrow \angle 1 = \angle 2$)

$\Rightarrow \angle 3 = \angle 4$

12. In the figure, line *l* and *m* intersect at *O*, forming angles as shown in the figure.



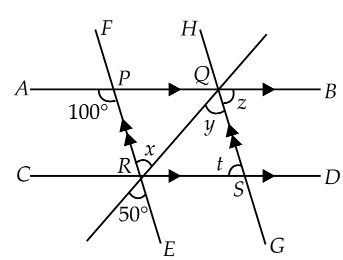
(i) If $x = 45°$, what is z?

(ii) If $v = 125°$, what is y?

Soln.: **(i)** $z = x = 45°$ (Vertically opposite angles)

(ii) $y = v = 125°$ (Vertically opposite angles)

13. In the given figure, *AB* $\parallel$ *CD* and *EF* $\parallel$ *GH*. Find the values of *x*, *y*, *z* and *t*.



Soln.: $EF \parallel GH$ and RQ is a transversal

$\Rightarrow y = 50°$ (Corresponding angles)

$\because EF$ and RQ interesect at R.

$\Rightarrow x = 50°$ (Vertically opposite angles)

Now, $EF \parallel GH$ and PQ is a transversal

$\Rightarrow 100° + z = 180°$ (Co-exterior angles)

$\Rightarrow z = 180° - 100°$

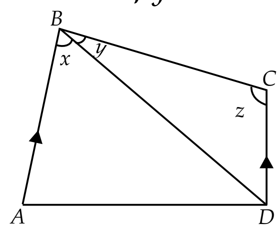
$\Rightarrow z = 80°$

$\because AB \parallel CD$ and QS is a transversal

$\Rightarrow \quad t = z$ (Alternate interior angles)

$\Rightarrow \quad t = 80°$

14. In the figure, $AB \parallel DC$. If $x = \frac{4}{3}y$ and $y = \frac{3}{8}z$ find the values of x, y and z.



Soln.: $AB \parallel DC$ and BC is a transversal

$\Rightarrow \quad (x + y) + z = 180°$ (co-interior angles)

$\Rightarrow \quad \frac{4}{3}y + y + \frac{8}{3}y = 180°$

$\left(\because \; x = \frac{4}{3}y \text{ and } y = \frac{3}{8}z \Rightarrow z = \frac{8}{3}y\right)$

$\Rightarrow \quad \frac{15y}{3} = 180° \quad \Rightarrow \quad y = \frac{180° \times 3}{15}$

$\Rightarrow \quad y = 36°$

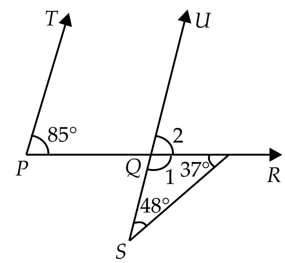
Now, $x = \frac{4}{3}y$

$\Rightarrow \quad x = \frac{4}{3} \times 36° \quad \Rightarrow \quad x = 48°$

and, $z = \frac{8}{3}y$

$\Rightarrow \quad z = \frac{8}{3} \times 36° \quad \Rightarrow \quad z = 96°$

15. In the figure, prove that $TP \parallel QU$.



Soln.: In ΔQRS

$\angle 2 = 48° + 37°$ (Exterior angle property of a triangle)

$\Rightarrow \quad \angle 2 = 85°$

$\Rightarrow \quad \angle UQR = 85°$

So, $\angle TPR = \angle UQR$

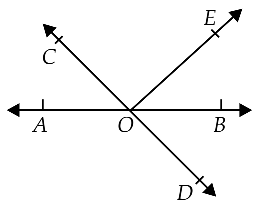
$\Rightarrow \quad TP \parallel UQ$

(As corresponding angles are equal)

NCERT SECTION

Exercise 6.1

1. In the given figure, lines AB and CD intersect at O. If $\angle AOC + \angle BOE = 70°$ and $\angle BOD = 40°$, find $\angle BOE$ and reflex $\angle COE$.



Soln.: Since AB is a straight line,

$\therefore \quad \angle AOC + \angle COE + \angle EOB = 180°$

or $\quad (\angle AOC + \angle BOE) + \angle COE = 180°$

or $\quad 70° + \angle COE = 180°$

$[\because \quad \angle AOC + \angle BOE = 70°$ (Given)]

or $\quad \angle COE = 180° - 70° = 110°$

$\therefore \quad$ Reflex $\angle COE = 360° - 110° = 250°$

Also, AB and CD intersect at O.

$\therefore \quad \angle COA = \angle BOD$ [Vertically opposite angles]

But $\angle BOD = 40°$ [Given]

$\therefore \quad \angle COA = 40°$

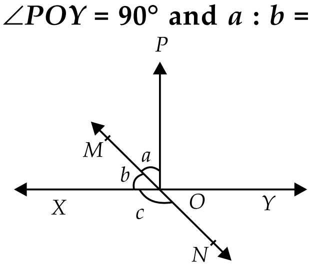
Also, $\angle AOC + \angle BOE = 70°$

$\therefore \quad 40° + \angle BOE = 70°$

or $\quad \angle BOE = 70° - 40° = 30°$

Thus, $\angle BOE = 30°$ and reflex $\angle COE = 250°$.

2. In the given figure, lines XY and MN intersect at O. If $\angle POY = 90°$ and $a : b = 2 : 3$, find c.



Soln.: Since XOY is a straight line.

$\therefore \quad \angle b + \angle a + \angle POY = 180°$

But $\angle POY = 90°$ [Given]

$\therefore \quad \angle b + \angle a = 180° - 90° = 90°$

Also $a : b = 2 : 3$

$$a = \left[\frac{90°}{2+3}\right] \times 2 = \frac{90°}{5} \times 2 = 36°$$

$$b = \frac{90°}{5} \times 3 = 54°$$

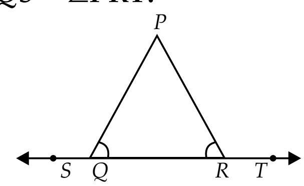
Since XY and MN intersect at O,

$\therefore c = [a + \angle POY]$ [Vertically opposite angles]

or $\quad c = 36° + 90° = 126°$

Thus, the required measure of $c = 126°$.

3. In the given figure, $\angle PQR = \angle PRQ$, then prove that $\angle PQS = \angle PRT$.



Soln.: ST is a straight line,

$\therefore \angle PQR + \angle PQS = 180°$ ...(1) [Linear pair]

Similarly, $\angle PRT + \angle PRQ = 180°$ ...(2) [Linear Pair]

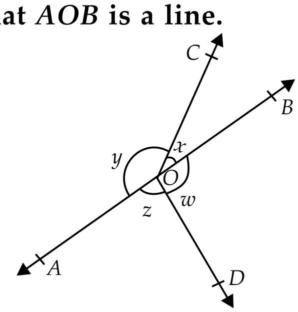
From (1) and (2), we have

$\angle PQS + \angle PQR = \angle PRT + \angle PRQ$

But $\angle PQR = \angle PRQ$ [Given]

$\therefore \angle PQS = \angle PRT$

4. In the given figure, if $x + y = w + z$, then prove that AOB is a line.



Soln.: Sum of all the angles at a point = 360°

$\therefore x + y + z + w = 360°$

or, $\quad (x + y) + (z + w) = 360°$

But $(x + y) = (z + w)$ [Given]

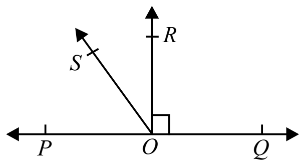
$\therefore (x + y) + (x + y) = 360°$

or, $\quad 2(x + y) = 360°$

or, $\quad (x + y) = \frac{360°}{2} = 180°$

$\therefore AOB$ is a straight line.

5. In the given figure, POQ is a line. Ray OR is perpendicular to line PQ. OS is another ray lying between rays OP and OR. Prove that $\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$.



Soln.: POQ is a straight line. [Given]

$\therefore \angle POS + \angle ROS + \angle ROQ = 180°$

But $OR \perp PQ$ $\quad\therefore \angle ROQ = 90°$

$\Rightarrow \angle POS + \angle ROS + 90° = 180°$

$\Rightarrow \angle POS + \angle ROS = 90°$

$\Rightarrow \angle ROS = 90° - \angle POS$ ... (1)

Now, we have $\angle ROS + \angle ROQ = \angle QOS$

$\Rightarrow \angle ROS + 90° = \angle QOS$

$\Rightarrow \angle ROS = \angle QOS - 90°$ ... (2)

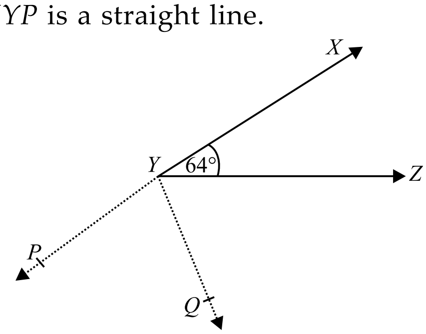
Adding (1) and (2), we have

$\Rightarrow 2\angle ROS = (\angle QOS - \angle POS)$

$\therefore \angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$.

6. It is given that $\angle XYZ = 64°$ and XY is produced to point P. Draw a figure from the given information. If ray YQ bisects $\angle ZYP$, find $\angle XYQ$ and reflex $\angle QYP$.

Soln.: XYP is a straight line.



$\therefore \angle XYZ + \angle ZYQ + \angle QYP = 180°$

$\Rightarrow 64° + \angle ZYQ + \angle QYP = 180°$

[$\because \angle XYZ = 64°$ (given)]

$\Rightarrow 64° + 2\angle QYP = 180°$

[YQ bisects $\angle ZYP$ so, $\angle QYP = \angle ZYQ$]

$\Rightarrow 2\angle QYP = 180° - 64° = 116°$

$\Rightarrow \angle QYP = \frac{116°}{2} = 58°$

$\therefore$ Reflex $\angle QYP = 360° - 58° = 302°$

Since $\angle XYQ = \angle XYZ + \angle ZYQ$

$\Rightarrow \angle XYQ = 64° + \angle QYP$

[$\because \angle XYZ = 64°$(Given) and $\angle ZYQ = \angle QYP$]

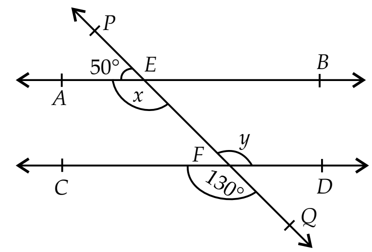
$\Rightarrow \angle XYQ = 64° + 58° = 122°$ [$\angle QYP = 58°$]

Thus,

$\angle XYQ = 122°$ and reflex $\angle QYP = 302°$.

Exercise 6.2

1. In the given figure, find the values of x and y and then show that $AB \parallel CD$.



Soln.: In the figure, we have CD and PQ intersect at F.

$\therefore y = 130°$ ...(1)

[Vertically opposite angles]

Again, PQ is a straight line and EA stands on it.

$\therefore \angle AEP + \angle AEQ = 180°$ [Linear pair]

or $50° + x = 180°$

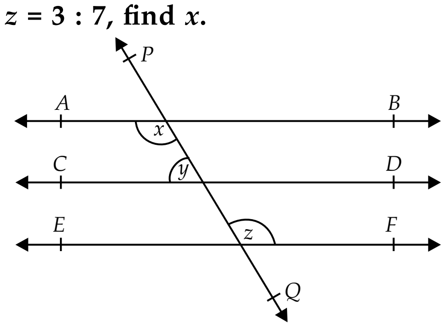
$\Rightarrow x = 180° - 50° = 130°$ ... (2)

From (1) and (2), $x = y$

As they are pair of alternate interior angles.

$\therefore AB \parallel CD$

2. In the given figure, if $AB \parallel CD$, $CD \parallel EF$ and $y : z = 3 : 7$, find x.



Soln.: $AB \parallel CD$ and $EF \parallel CD$ [Given]

$\therefore AB \parallel EF$ and PQ is a transversal.

$\therefore x = z$ ... (1)

[Alternate interior angles]

Again, $AB \parallel CD$

$\Rightarrow x + y = 180°$ [Co-interior Angles]

$\Rightarrow z + y = 180°$ [By (1)]

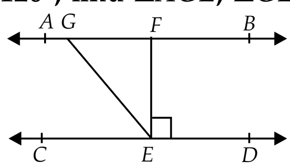
But $y : z = 3 : 7$

$$\therefore\ z = \left[\frac{180°}{(3+7)}\right] \times 7 = \frac{180°}{10} \times 7 = 126° \qquad \dots (2)$$

From (1) and (2), we have

$x = 126°$.

3. In the given figure, if $AB \parallel CD$, $EF \perp CD$ and $\angle GED = 126°$, find $\angle AGE$, $\angle GEF$ and $\angle FGE$.



Soln.: $AB \parallel CD$ and GE is a transversal.

$\therefore \angle AGE = \angle GED$

[Alternate interior angles]

But $\angle GED = 126°$ [Given]

$\therefore \angle AGE = 126°$

Also, $\angle GEF + \angle FED = \angle GED$

or $\angle GEF + 90° = 126°$

[$\because$ $EF \perp CD$ (given)]

$\angle GEF = 126° - 90° = 36°$

Now, $AB \parallel CD$ and GE is a transversal.

$\therefore \angle FGE + \angle GED = 180°$

[Co-interior angles]

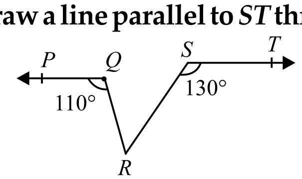
or $\angle FGE + 126° = 180°$

or $\angle FGE = 180° - 126° = 54°$

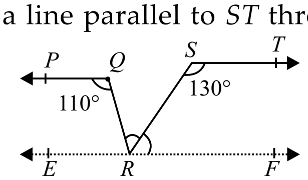
Thus, $\angle AGE = 126°$, $\angle GEF = 36°$ and $\angle FGE = 54°$.

4. In the given figure, if $PQ \parallel ST$, $\angle PQR = 110°$ and $\angle RST = 130°$, find $\angle QRS$.

[Hint: Draw a line parallel to ST through point R.]



Soln.: Draw a line parallel to ST through R.



Since $PQ \parallel ST$ [Given]

and $EF \parallel ST$ [Construction]

$\therefore$ $PQ \parallel EF$ and QR is a transversal

$\Rightarrow$ $\angle PQR = \angle QRF$

[Alternate Interior Angles]

But $\angle PQR = 110°$ [Given]

$\therefore \angle QRF = \angle QRS + \angle SRF = 110°$ ...(1)

Again $ST \parallel EF$ and RS is a transversal

$\therefore \angle RST + \angle SRF = 180°$

[Co-interior angles]

or $130° + \angle SRF = 180°$

$\Rightarrow$ $\angle SRF = 180° - 130° = 50°$

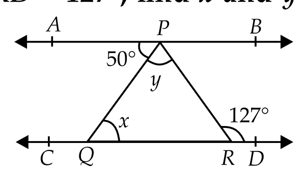
Now, from (1), we have

$\angle QRS + 50° = 110°$

$\Rightarrow$ $\angle QRS = 110° - 50° = 60°$

Thus, $\angle QRS = 60°$.

5. In the given figure, if $AB \parallel CD$, $\angle APQ = 50°$ and $\angle PRD = 127°$, find x and y.



Soln.: We have $AB \parallel CD$ and PQ is a transversal.

$\therefore \angle APQ = \angle PQR$

[Alternate interior angles]

or $50° = x$ [$\because$ $\angle APQ = 50°$ (Given)]

Again, $AB \parallel CD$ and PR is a transversal.

$\therefore \angle APR = \angle PRD$

[Alternate interior angles]

$\Rightarrow$ $\angle APR = 127°$

[$\because$ $\angle PRD = 127°$ (given)]

$\Rightarrow$ $\angle APQ + \angle QPR = 127°$

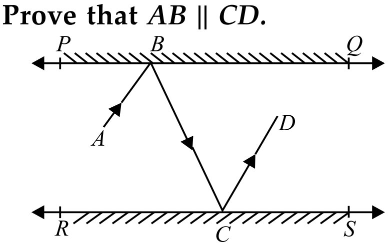
$\Rightarrow$ $50° + y = 127°$

[$\because$ $\angle APQ = 50°$ (given)]

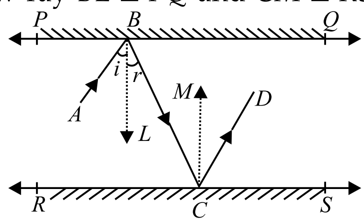
$\Rightarrow$ $y = 127° - 50° = 77°$

Thus, $x = 50°$ and $y = 77°$.

6. In the given figure, PQ and RS are two mirrors placed parallel to each other. An incident ray AB strikes the mirror PQ at B, the reflected ray moves along the path BC and strikes the mirror RS at C and again reflects back along CD. Prove that $AB \parallel CD$.



Soln.: Draw ray $BL \perp PQ$ and $CM \perp RS$



$\because PQ \parallel RS \Rightarrow BL \parallel CM$

$[\because BL \perp PQ$ and $CM \perp RS]$

Now, $BL \parallel CM$ and BC is a transversal.

$\therefore \angle LBC = \angle MCB$ ...(1)

[Alternate interior angles]

Since, angle of incidence = Angle of reflection

$\angle ABL = \angle LBC$ and $\angle MCB = \angle MCD$

$\Rightarrow \angle ABL = \angle MCD$ ...(2)

$\therefore$ Adding (1) and (2), we have

$\angle LBC + \angle ABL = \angle MCB + \angle MCD$

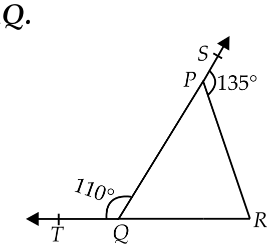
$\Rightarrow \angle ABC = \angle BCD$

i.e., a pair of alternate interior angles are equal,

$\therefore AB \parallel CD$.

Exercise 6.3

1. In the adjoining figure, sides QP and RQ of ΔPQR are produced to points S and T respectively. If $\angle SPR = 135°$ and $\angle PQT = 110°$, find $\angle PRQ$.



Soln.: We have, $\angle TQP + \angle PQR = 180°$ [Linear pair]

$\Rightarrow 110° + \angle PQR = 180°$

$\Rightarrow \angle PQR = 180° - 110° = 70°$

Since, the side QP of ΔPQR is produced to S.

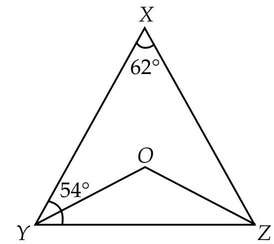
$\Rightarrow \angle PQR + \angle PRQ = 135°$

[Exterior angle property of a Δ]

$\Rightarrow 70° + \angle PRQ = 135°$ $[\angle PQR = 70°]$

$\Rightarrow \angle PRQ = 135° - 70° \Rightarrow \angle PRQ = 65°$

2. In the adjoining figure, $\angle X = 62°$, $\angle XYZ = 54°$. If YO and ZO are the bisectors of $\angle XYZ$ and $\angle XZY$ respectively of ΔXYZ, find $\angle OZY$ and $\angle YOZ$.



Soln.: In ΔXYZ, we have

$\angle XYZ + \angle YZX + \angle ZXY = 180°$

[Angle sum property of a triangle]

But $\angle XYZ = 54°$ and $\angle ZXY = 62°$

$\therefore 54° + \angle YZX + 62° = 180°$

$\Rightarrow \angle YZX = 180° - 54° - 62° = 64°$

YO and ZO are the bisectors of $\angle XYZ$ and $\angle XZY$ respectively,

$\therefore \angle OYZ = \frac{1}{2}\angle XYZ = \frac{1}{2}(54°) = 27°$

and $\angle OZY = \frac{1}{2}\angle YZX = \frac{1}{2}(64°) = 32°$

Now, in ΔOYZ, we have

$\angle YOZ + \angle OYZ + \angle OZY = 180°$

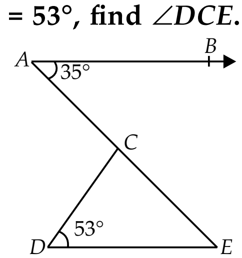
[Angle sum property of a triangle]

$\Rightarrow \angle YOZ + 27° + 32° = 180°$

$\Rightarrow \angle YOZ = 180° - 27° - 32° = 121°$

Thus, $\angle OZY = 32°$ and $\angle YOZ = 121°$

3. In the given figure, if $AB \parallel DE$, $\angle BAC = 35°$ and $\angle CDE = 53°$, find $\angle DCE$.



Soln.: $AB \parallel DE$ and AE is a transversal.

So, $\angle BAC = \angle AED$

[Alternate interior angles]

and $\angle BAC = 35°$ [Given]

$\therefore \angle AED = 35°$

Now, in ΔCDE, we have

$\angle CDE + \angle DEC + \angle DCE = 180°$

[Angle sum property of a triangle]

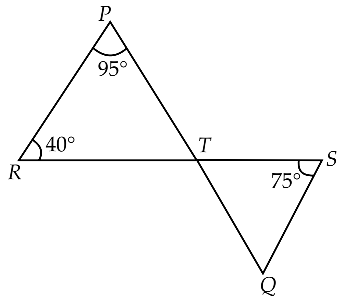
$\therefore 53° + 35° + \angle DCE = 180°$

$[\because \angle DEC = \angle AED = 35°$ and $\angle CDE = 53°$ (Given)$]$

$\Rightarrow \angle DCE = 180° - 53° - 35° = 92°$

Thus, $\angle DCE = 92°$

4. **In the adjoining figure, if lines PQ and RS intersect at point T, such that $\angle PRT = 40°$, $\angle RPT = 95°$ and $\angle TSQ = 75°$, find $\angle SQT$.**



Soln.: In ΔPRT, we have

$\angle P + \angle R + \angle PTR = 180°$

[Angle sum property of a triangle]

$\Rightarrow \quad 95° + 40° + \angle PTR = 180°$

[$\because \ \angle P = 95°, \angle R = 40°$ (given)]

$\Rightarrow \quad \angle PTR = 180° - 95° - 40° = 45°$

But PQ and RS intersect at T,

$\therefore \angle PTR = \angle QTS$

[Vertically opposite angles]

$\therefore \angle QTS = 45°$ [$\because \ \angle PTR = 45°$]

Now, in ΔTQS, we have

$\angle TSQ + \angle STQ + \angle SQT = 180°$

[Angle sum property of a triangle]

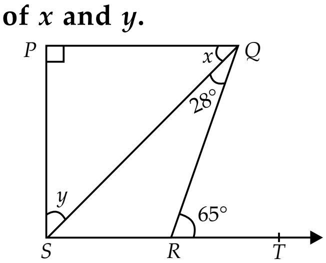
$\therefore 75° + 45° + \angle SQT = 180°$

[$\because \ \angle TSQ = 75°$ and $\angle STQ = 45°$]

$\Rightarrow \quad \angle SQT = 180° - 75° - 45° = 60°$

Thus, $\angle SQT = 60°$

5. **In the adjoining figure, if $PQ \perp PS$, $PQ \parallel SR$, $\angle SQR = 28°$ and $\angle QRT = 65°$, then find the values of x and y.**



Soln.: In ΔQRS, the side SR is produced to T.

$\therefore \angle QRT = \angle RQS + \angle RSQ$

[Exterior angle property of a triangle]

But $\angle RQS = 28°$ and $\angle QRT = 65°$

So, $\ 28° + \angle RSQ = 65°$

$\Rightarrow \quad \angle RSQ = 65° - 28° = 37°$

Since, $PQ \parallel SR$ and QS is a transversal.

$\therefore \angle PQS = \angle RSQ = 37°$

[Alternate interior angles]

$\Rightarrow \quad x = 37°$

Again, $PQ \perp PS \ \Rightarrow \ \angle P = 90°$

Now, in ΔPQS, we have

$\angle P + \angle PQS + \angle PSQ = 180°$

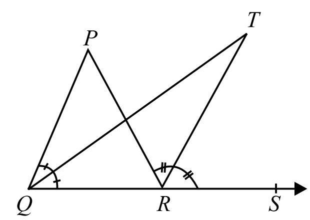
[Angle sum property of a triangle]

$\Rightarrow \quad 90° + 37° + y = 180°$

$\Rightarrow \quad y = 180° - 90° - 37° = 53°$

Thus, $\ x = 37°$ and $y = 53°$

6. **In the adjoining figure, the side QR of ΔPQR is produced to a point S. If the bisectors of $\angle PQR$ and $\angle PRS$ meet at point T, then prove that $\angle QTR = \frac{1}{2}\angle QPR$.**



Soln.: In ΔPQR, side QR is produced to S, so by exterior angle property,

$\angle PRS = \angle P + \angle PQR$

$$\Rightarrow \frac{1}{2}\angle PRS = \frac{1}{2}\angle P + \frac{1}{2}\angle PQR \qquad ...(1)$$

$$\Rightarrow \angle TRS = \frac{1}{2}\angle P + \angle TQR \qquad ...(2)$$

[$\because$ QT and RT are bisectors of $\angle PQR$ and $\angle PRS$ respectively.]

Now, in ΔQRT, we have

$\angle TRS = \angle TQR + \angle T \qquad ...(3)$

[Exterior angle property of a triangle]

From (2) and (3), we have

$$\angle TQR + \frac{1}{2}\angle P = \angle TQR + \angle T$$

$$\Rightarrow \frac{1}{2}\angle P = \angle T \quad \Rightarrow \frac{1}{2}\angle QPR = \angle QTR$$

$$\text{or} \quad \angle QTR = \frac{1}{2}\angle QPR$$

EXERCISE

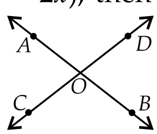
Multiple Choice Questions

Level-1

1. An angle is 18° less than its complementary angle. The measure of this angle is
(a) 36° (b) 48° (c) 83° (d) 81°

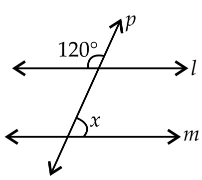
2. Supplement of an angle is one fourth of itself. The measure of the angle is
(a) 18° (b) 36° (c) 144° (d) 72°

3. Line AB and CD intersect to O. If $\angle AOC = (3x - 10°)$ and $\angle BOD = (20° - 2x)$, then the value of x, is



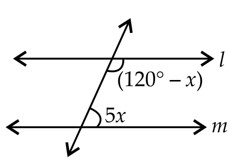
(a) 6° (b) 12° (c) 36° (d) 30°

4. If $l \parallel m$, then value of x is



(a) 60° (b) 120°
(c) 40°
(d) Cannot be determined

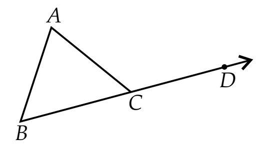
5. The value of x from the adjoining figure, if $l \parallel m$ is



(a) 15° (b) 10° (c) 19° (d) 36°

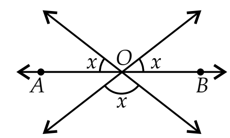
6. In ΔABC, $\angle A : \angle B : \angle C = 2 : 3 : 5$, then angle at B is
(a) 54° (b) 126°
(c) 136° (d) 64°

7. In adjoining figure if $\angle A = (3x + 2°)$, $\angle B = (x - 3°)$, $\angle ACD = 127°$, then $\angle A$ =



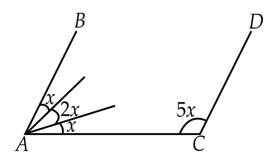
(a) 24° (b) 32° (c) 96° (d) 98°

8. The value of x if AOB is a straight line, is



(a) 36° (b) 60° (c) 30° (d) 35°

9. If $AB \parallel CD$, what is the value of x?



(a) 18° (b) 15° (c) 20° (d) 25°

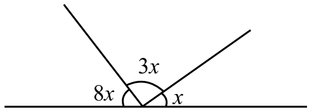
10. If two parallel lines are intersected by a transversal, then each pair of corresponding angles so formed is
(a) Equal (b) Complementary
(c) Supplementry (d) None of these

11. An angle is 14° more than its complementary angle, then angle is
(a) 30° (b) 52°
(c) 50° (d) None of these

12. If the supplement of an angle is three times its complement, then angle is
(a) 40° (b) 35°
(c) 50° (d) 45°

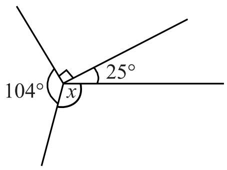
13. Which one of the following statement is not false ?
(a) If two angles forming a linear pair, then each of these angle is of measure 90°.
(b) Angles forming a linear pair can both be acute angles.
(c) Both of the angles forming a linear pair can be obtuse angles.
(d) Bisectors of the adjacent angles forming a linear pair form a right angle.

14. Calculate the value of x.



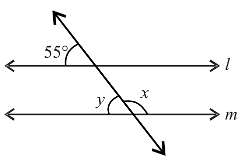
(a) 270° (b) 70° (c) 15° (d) 45°

15. Calculate the value of x.



(a) 141° (b) 70° (c) 105° (d) 45°

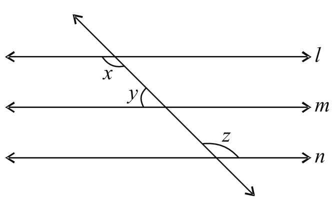
16. In figure, if $l \parallel m$, then x =



(a) 145° (b) 125°
(c) 115° (d) 140°

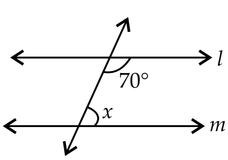
17. In Q.16, value of y =
(a) 45° (b) 65°
(c) 55° (d) 82°

18. In figure, if $l \parallel m$, $l \parallel n$ and $x : y = 3 : 2$, then z =



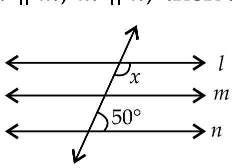
(a) 120° (b) 126°
(c) 108° (d) 154°

19. In figure, if $l \parallel m$, then x =



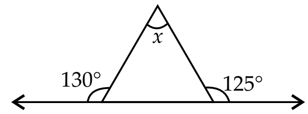
(a) 120° (b) 110°
(c) 90° (d) 98°

20. In figure, if $l \parallel m$, $m \parallel n$, then x =



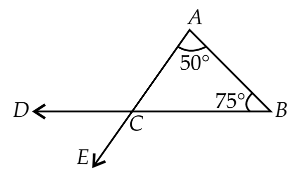
(a) 130° (b) 140°
(c) 120° (d) 154°

21. Find the value of x.



(a) 70° (b) 75° (c) 60° (d) 65°

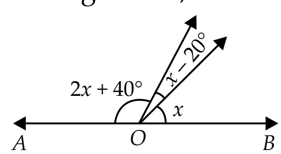
22. Find $\angle DCE$.



(a) 45° (b) 60° (c) 50° (d) 55°

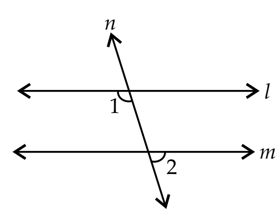
23. If two interior angles on the same side of a transversal intersecting two parallel lines are in the ratio 5 : 4, then the greater of the two angles is
(a) 54° (b) 100° (c) 120° (d) 136°

24. If AOB is a straight line, then x is



(a) 60° (b) 30° (c) 90° (d) 40°

25. In the adjoining figure, if $l \parallel m$ and n be the transversal, then the relation between $\angle 1$ and $\angle 2$ is



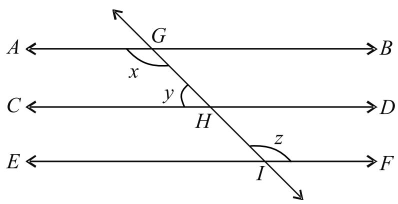
(a) $\angle 1 = \angle 2$ (b) $\angle 1 + \angle 2 = 180°$
(c) $\angle 1 - \angle 2 = 90°$ (d) $\angle 1 + \angle 2 = 90°$

Level-2

26. If angle with measure x and y form a complementary pair, then angles with which of the following measures will form a supplementary pair ?
(a) $(x + 47°)$, $(y + 43°)$ (b) $(x - 23°)$, $(y + 23°)$
(c) $(x - 43°)$, $(y - 47°)$
(d) No such pair is possible

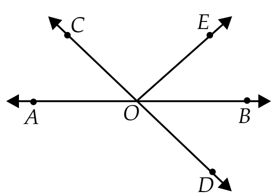
27. If one angle of a triangle is equal to the sum of the other two angles, then triangle is a/an
(a) acute angled triangle
(b) obtuse angled triangle
(c) right angled triangle
(d) none of these

28. In figure, if $AB \parallel CD$, $CD \parallel EF$ and $y : z = 4 : 5$, $\angle x$ =



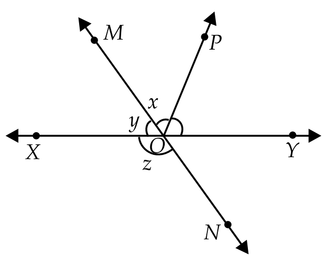
(a) 100° (b) 76° (c) 82° (d) 122°

29. In figure, lines AB and CD intersect at O. If $\angle AOC + \angle BOE = 100°$ and $\angle BOD = 60°$, find $\angle BOE$ and reflex $\angle COE$ respectively.



(a) 40°, 280° (b) 40°, 260°
(c) 30°, 260° (d) 30°, 250°

30. In figure, lines XY and MN intersect at O. If $\angle POY = 70°$ and $x : y = 3 : 2$, find z.



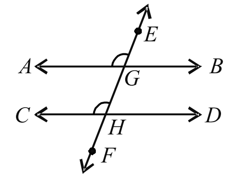
(a) 70° (b) 95° (c) 136° (d) 120°

Fill in the Blanks

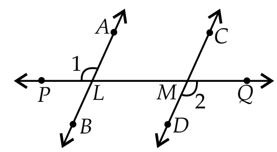
1. If a transversal intersects two parallel lines, then the sum of the interior angles on the same side of the transversal is ....

2. Lines which are parallel to the same line are .... to each other.

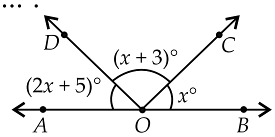
3. In figure, $AB \parallel CD$ and transversal EF cuts them at G and H respectively. If $\angle AGE = 110°$, then $\angle GHD$ = ...



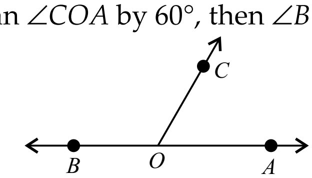
4. In figure, a transversal PQ intersects two parallel lines AB and CD at L and M respectively. If $\angle 1 = 95°$, then $\angle 2$ = ...



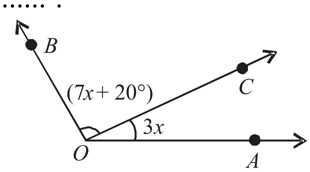
5. In figure, if AOB is a straight line, then $\angle BOC$ = ...



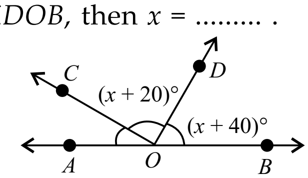
6. In figure, BOA is a straight line and $\angle BOC$ is greater than $\angle COA$ by 60°, then $\angle BOC$ = ...



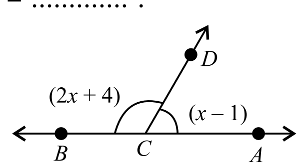
7. In figure, if $\angle BOC = 7x + 20°$ and $\angle COA = 3x$, the magnitude of x which makes AOB a straight line is ....



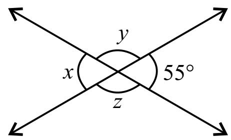
8. In figure, AOB is a straight line, if $\angle COA = \angle DOB$, then x = ...



9. In figure, x = ...



10. In figure, $\angle x$ = ..., $\angle y$ = ..., $\angle z$ = ...



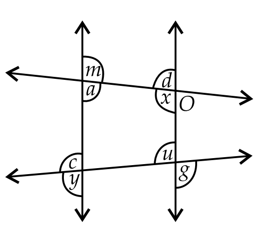
True or False

1. Angles forming a linear pair are supplementary.
2. If two adjacent angles are equal, then each angle measure 90°.
3. Angles forming a linear pair can be both acute angles.
4. If angles forming a linear pair are equal, then each of these angles is of measure 90°.
5. If two lines intersect each other and one pair of vertically opposite angles is formed by acute angles, then the other pair of vertically opposite angles will be formed by obtuse angles.
6. If two lines are intersected by a transversal, then corresponding angles are equal.
7. If two parallel lines are intersected by a transversal, then alternate angles are equal.
8. If two lines are intersected by a transversal, then angles on the same side of transversal are supplementary.
9. Two angles are complementary if their sum is 90°.
10. The supplementary angles have their sum equal to 360°.

Match the Following

In this section each question has two matching lists. Choices for the correct combination of elements from List-I and List-II are given as options (a), (b), (c) and (d) out of which one is correct.

1. Use the given figure to match List-I with List-II.

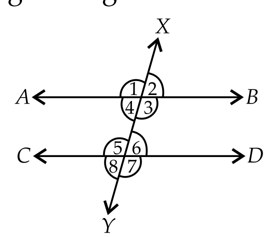


	List-I		List-II
(P)	Angles m and y are	(1)	Alternate interior pair of angles
(Q)	Angles a and d are	(2)	Alternate exterior pair of angles
(R)	Angles d and u are	(3)	Vertically opposite angles
(S)	Angles u and g are	(4)	Corresponding angles

Code :

	P	Q	R	S
(a)	3	1	2	4
(b)	2	1	4	3
(c)	4	2	1	3
(d)	1	4	3	2

2. Use the given figure to match List-I with List-II.



, $AB \parallel CD$

	List-I		List-II
(P)	Corresponding angles	(1)	$\angle 1 = \angle 7$
(Q)	Alternate interior angles	(2)	$\angle 4 + \angle 5 = 180°$
(R)	Alternate exterior angles	(3)	$\angle 1 = \angle 5$
(S)	Co-interior angles	(4)	$\angle 4 = \angle 6$

Code :

	P	Q	R	S
(a)	4	1	2	3
(b)	3	2	4	1
(c)	4	2	1	3
(d)	3	4	1	2

Assertion & Reason Type

Directions : In each of the following questions, a statement of Assertion is given followed by a corresponding statement of Reason just below it. Of the statements, mark the correct answer as

(a) *If both assertion and reason are true and reason is the correct explanation of assertion.*
(b) *If both assertion and reason are true but reason is not the correct explanation of assertion.*
(c) *If assertion is true but reason is false.*
(d) *If assertion is false but reason is true.*

1. **Assertion :** Two adjacent angles always form a linear pair.

Reason : In a linear pair of angles two non-common arms are opposite rays.

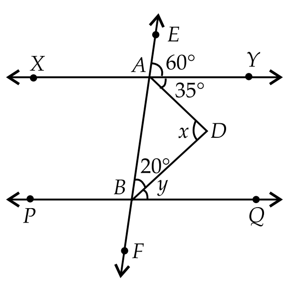
2. **Assertion :** The bisectors of the angles of a linear pair are at right angles.

Reason : If the sum of two adjacent angles is 180°, then the non-common arms of the angles are in a straight line.

3. **Assertion :** If a line is perpendicular to one of the two given parallel lines then it is also perpendicular to the other line.

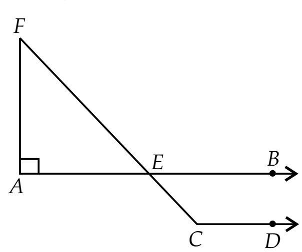
Reason : If two lines are intersected by a transversal then the bisectors of any pair of alternate interior angles are parallel.

4. **Assertion :** In figure, if XY is parallel to PQ, then the angles x and y are 70° and 45° respectively.



Reason : Sum of angles of a triangle is 180°.

5. **Assertion :** In the given figure, if $AB \parallel CD$ and $\angle F = 30°$, then $\angle FCD$ is 120°.



Reason : If two parallel lines are intersected by a transversal, then co-interior angles are equal.

Comprehension Type

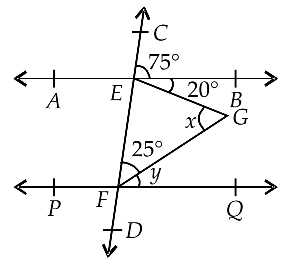
PASSAGE-I : If a transversal intersects two parallel lines, then

(i) each pair of corresponding angles are equal

(ii) each pair of alternate interior angles are equal

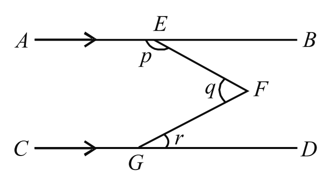
(iii) each pair of co-interior angles are supplementary.

1. In the given figure, $AB \parallel PQ$. The values of x and y respectively are



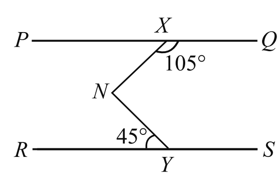
(a) 50°, 70° (b) 70°, 50°
(c) 75°, 45° (d) 20°, 75°

2. In the figure, $AB \parallel CD$. Then the value of $p + q - r =$



(a) 80° (b) 180°
(c) 100° (d) 360°

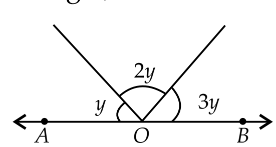
3. In the given figure, $PQ \parallel RS$ and $\angle QXN = 105°$, $\angle RYN = 45°$, find $\angle XNY$.



(a) 45° (b) 30°
(c) 60° (d) 120°

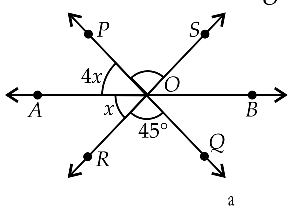
PASSAGE-II : The sum of all the angles formed on the same side of a line at a given point on the line is 180°.

1. Find largest angle, if AOB is a straight line.



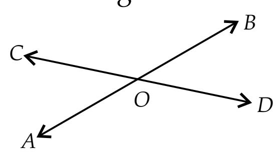
(a) 60° (b) 30°
(c) 80° (d) 90°

2. If three straight lines AB, PQ and RS intersect at O. Find the value of x from the given figure.



(a) 20° (b) 17° (c) 27° (d) 54°

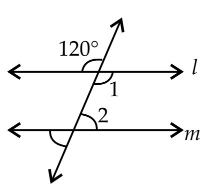
3. In the given figure AB and CD are two lines intersecting at O. If $\angle AOC$ and $\angle BOC$ are in ratio 2 : 3, find all angles.



(a) 72°, 108°, 108°, 72°
(b) 45°, 100°, 100°, 45°
(c) 30°, 120°, 30°, 120°
(d) 50°, 120°, 50°, 120°

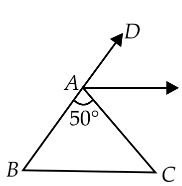
PASSAGE-III : If a transversal intersects two lines in such a way that a pair of alternate interior angles are equal, or each pair of co-interior angles are supplementary then the two lines are parallel.

1. From the given figure for what value of $\angle 2$ if $l \parallel m$.



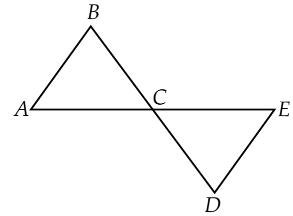
(a) 120° (b) 180° (c) 60° (d) 100°

2. In the given figure, ABC is an isosceles triangle with $AB = AC$ and $AP \parallel BC$, $\angle BAC = 50°$. Find $\angle DAP$



(a) 50° (b) 60° (c) 65° (d) 70°

3. In the given figure $AB \parallel DE$. If $\angle B = 45°$ and $\angle DEC = 75°$. Find $\angle ACD$.

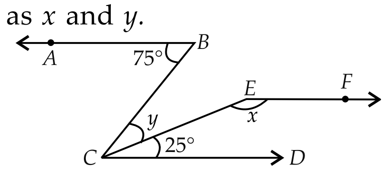


(a) 60° (b) 90° (c) 150° (d) 120°

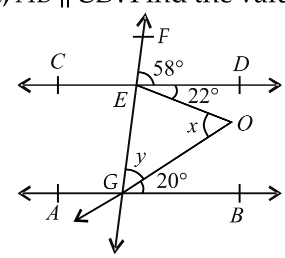
Subjective Problems

Very Short Answer Type

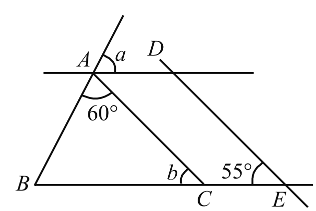
1. In the figure $AB \parallel CD \parallel EF$, find the angles marked as x and y.



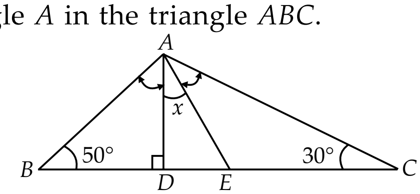
2. In the figure, $AB \parallel CD$. Find the values of x and y.



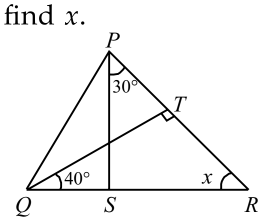
3. In the figure $AD \parallel BE$ and $AC \parallel DE$, find a and b.



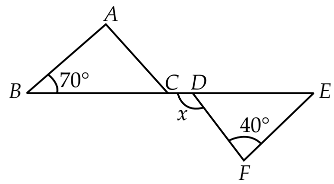
4. In the figure, find the value of x, if AE is bisector of angle A in the triangle ABC.



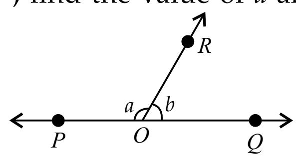
5. In figure, if $QT \perp PR$, $\angle TQR = 40°$ and $\angle SPR = 30°$, find x.



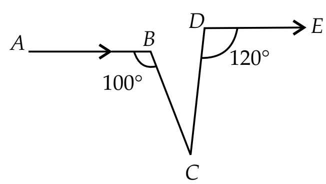
6. In figure, $AB \parallel EF$, $\angle ABC = 70°$ and $\angle EFD = 40°$, then find x.



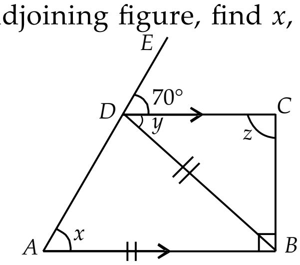
7. In figure, $\angle POR$ and $\angle QOR$ form a linear pair. If $a - b = 80°$, find the value of a and b.



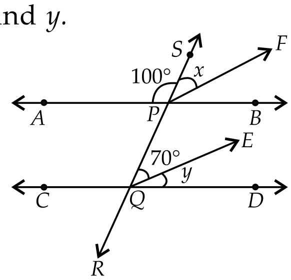
8. In the figure, $AB \parallel DE$. Find the value of $\angle BCD$.



9. From the adjoining figure, find x, y and z.

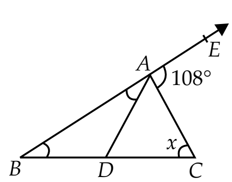


10 . In the given figure, $AB \parallel CD$, $PF \parallel QE$. Find value of x and y.

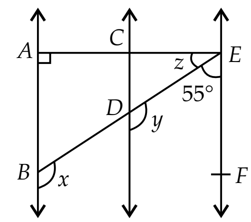


Short Answer Type

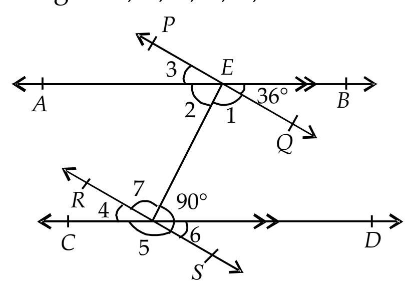
1. In the given figure, AD divides $\angle BAC$ in the ratio 1 : 3 and $AD = DB$. Determine the value of x.



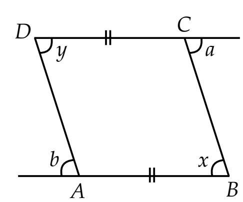
2. In figure, $AB \parallel CD$ and $CD \parallel EF$. Also $EA \perp AB$. If $\angle BEF = 55°$, find the values of x, y and z.



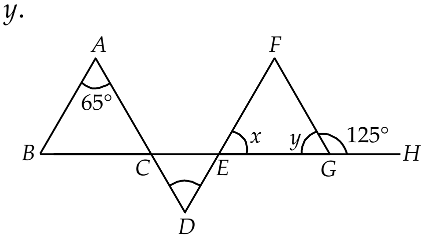
3. In the figure, $AB \parallel CD$ and $PQ \parallel RS$, find the value of angles 1, 2, 3, 4, 5, 6 and 7.



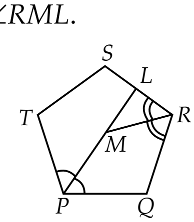
4. The sides BA and DC of the parallelogram $ABCD$ are produced as shown in figure. Prove that $a + b = x + y$.



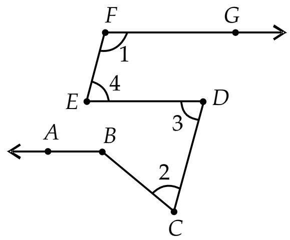
5. In the figure given, $AB \parallel DF$ and $AD \parallel FG$ find x and y.



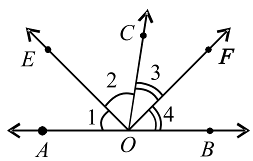
6. $PQRST$ is a regular pentagon and bisector of $\angle TPQ$ meets SR at L. If bisector of $\angle SRQ$ meets PL at M, find $\angle RML$.



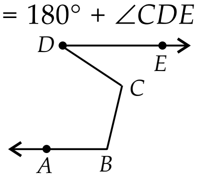
7. In figure, if $AB \parallel DE$, $DE \parallel FG$, $CD \parallel EF$, $\angle 2 = 55°$ and $\angle 4 = 60°$, then find $\angle 1$ and $\angle 3$.



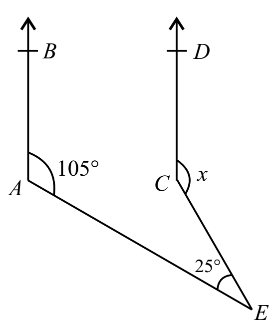
8. In the adjoining figure OE bisects $\angle AOC$, OF bisects $\angle COB$ and $OE \perp OF$. Show that A, O, B are collinear.



9. In figure, $AB \parallel DE$. Prove that $\angle ABC + \angle BCD = 180° + \angle CDE$.

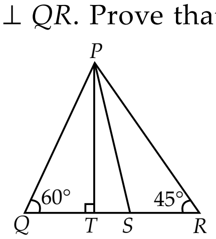


10. In the given figure, $AB \parallel CD$. Find the value of x.

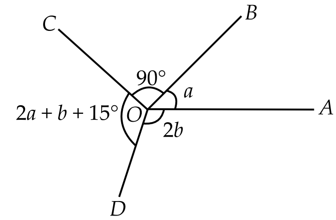


Long Answer Type

1. In the given figure, PS is the angle bisector of the $\angle QPR$. $PT \perp QR$. Prove that $\angle TPS = 7.5°$.

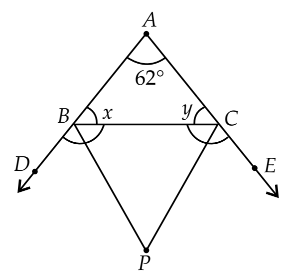


2. In the given figure, $2b - a = 65°$ and $\angle BOC = 90°$, find the measure of $\angle AOB$, $\angle AOD$ and $\angle COD$.

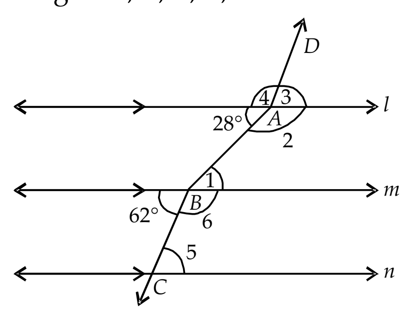


3. In a ΔABC, $\angle ABC = 90°$ and $BD \perp AC$. Prove that $\angle ABD = \angle ACB$.

4. In figure, the sides AB and AC of ΔABC are produced to points D and E respectively. If bisectors BP and CP of $\angle CBD$ and $\angle BCE$ respectively meet at point P, then find $\angle BPC$.



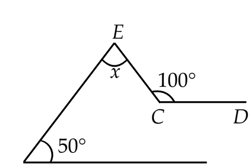
5. In the figure, $l \parallel m \parallel n$ and $AD \parallel BC$. Find the value of angles 1, 2, 3, 4, 5 and 6.



Integer Answer Type

In this section, each question, when worked out will result in one integer from 0 to 9 (both inclusive).

1. In the given figure, $AB \parallel CD$. Find the value of $\dfrac{x}{25°}$.

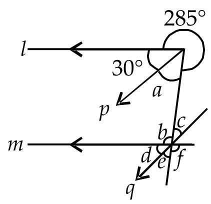


2. If an angle x is supplement of itself. Then value of $\dfrac{x - 60°}{6°}$ is

3. Angles of a triangle are in ratio 1 : 2 : 3, then the greatest angle is what times to the smallest angle.

4. If two interior angles on the same side of a transversal intersecting two parallel lines are in the ratio of 1 : 4. Then what will be the result if difference of the angles is divided by smaller angle.

Directions (5 – 10) : In the given figure, $l \parallel m$ and $p \parallel q$.



5. $\dfrac{a}{\circledcirc^\circ} =$

6. Angle b when divided by 21° is

7. What is the sum of digits of angle c?

8. What is the sum of digits of angle d?

9. Value of $\dfrac{e + 5^\circ}{10^\circ}$ is

10. Calculate $\dfrac{f - 5^\circ}{25^\circ}$.

CHAPTER

6 Lines and Angles

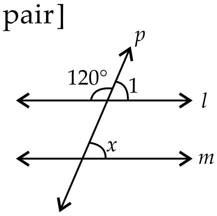
Multiple Choice Questions

1. **(a) :** Let the angle be x.
$\therefore$ its complement $= x + 18°$
$\Rightarrow x + x + 18° = 90° \Rightarrow 2x = 90° - 18°$
$\Rightarrow 2x = 72° \Rightarrow x = 36°$

2. **(c) :** Let the angle be x
$\therefore$ its supplement $= \frac{1}{4}$ of $x = \frac{1}{4}x$
$$\Rightarrow x + \frac{1}{4}x = 180° \Rightarrow \frac{5x}{4} = 180°$$
$$\Rightarrow x = \frac{180° \times 4}{5} = 144°$$

3. **(a) :** Since vertically opposite angles are always equal
$\therefore (3x - 10°) = (20° - 2x)$
$\Rightarrow 3x + 2x = 20° + 10°$
$\Rightarrow 5x = 30° \Rightarrow x = 6°$.

4. **(a) :** $\angle 1 + 120° = 180°$ [Linear pair]
$\Rightarrow \angle 1 = 180° - 120°$
$\Rightarrow \angle 1 = 60°$
Since $l \parallel m$



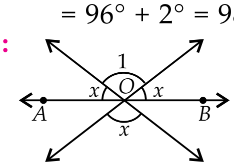
$\angle x = \angle 1 = 60°$ [Corresponding Angles]

5. **(a) :** Since $l \parallel m$
$\Rightarrow 120° - x + 5x = 180°$ [Co-interior angles]
$\Rightarrow 120° + 4x = 180°$
$\Rightarrow 4x = 60° \Rightarrow x = 15°$

6. **(a) :** $\angle A : \angle B : \angle C = 2 : 3 : 5$
$\Rightarrow \angle A = 2x, \ \angle B = 3x, \ \angle C = 5x$
$\therefore \angle A + \angle B + \angle C = 2x + 3x + 5x = 10x$
$\Rightarrow 10x = 180°$ [Angle sum property of a triangle]
$\Rightarrow x = 18°$
$\Rightarrow \angle B = 3 \times 18° = 54°$

7. **(d):** In ΔABC
$\angle ACD = \angle A + \angle B$
[By exterior angle property of a triangle]
$\Rightarrow 127° = 3x + 2° + x - 3° \Rightarrow 127° = 4x - 1°$
$\Rightarrow 128° = 4x$
$\Rightarrow x = 32°$
$\therefore \angle A = (3x + 2°) = 3 \times 32° + 2°$
$= 96° + 2° = 98°$

8. **(b):**



$\angle 1 = x$ [Vertically opposite angles]
Since AOB is a straight line
$\Rightarrow x + x + x = 180° \Rightarrow 3x = 180°$
$\therefore x = 60°$

9. **(c) :** Since $AB \parallel CD$
$\Rightarrow x + 2x + x + 5x = 180°$ [Co-interior angles]
$\Rightarrow 9x = 180°$
$\therefore x = 20°$

10. **(a)**

11. **(b):** Let the angle be x.
Complement of $x = (90° - x)$
Since the difference is 14°, we have
$x - (90° - x) = 14°$
$\Rightarrow 2x = 104° \Rightarrow x = 52°$.

12. **(d):** Let the angle be x.
Complement of $x = 90° - x$
Supplement of $x = 180° - x$
Given that, $180° - x = 3\,(90° - x)$
$\Rightarrow 180° - x = 270° - 3x$
$\Rightarrow 2x = 270° - 180°$
$\Rightarrow 2x = 90° \Rightarrow x = 45°$.

13. (d)

14. (c) : $8x + 3x + x = 180°$ [Linear Pair]

$12x = 180° \Rightarrow x = 15°$.

15. (a) : $104° + 90° + 25° + x = 360°$

[Complete Angle]

$\Rightarrow x = 141°$.

16. (b): Since $l \parallel m$, then

$y = 55°$ [Corresponding Angles]

Now, $x + y = 180°$ [Linear Pair]

$\Rightarrow x + 55° = 180°$

$\Rightarrow x = 125°$

17. (c)

18. (c) : We have, $l \parallel m$, $l \parallel n \Rightarrow m \parallel n$.

Now, $x : y = 3 : 2$

$\Rightarrow x = \frac{3}{2}y$ ... (i)

Also $x + y = 180°$ [Co-interior Angles]

$\Rightarrow \frac{3}{2}y + y = 180° \Rightarrow y = 72°$

Also, $x = z$ ...(ii) [Alternate Interior Angles]

From (i) and (ii), we have

$z = \frac{3}{2}y \Rightarrow z = \frac{3}{2} \times 72° = 108°$

19. (b): Since $l \parallel m$,

$70° + x = 180°$ [Co-interior Angles]

$\Rightarrow x = 110°$

20. (a) : Since $l \parallel m$ and $m \parallel n$, then $l \parallel n$

$\Rightarrow x + 50° = 180°$ [Co-interior Angles]

$\Rightarrow x = 130°$.

21. (b): $x + (180° - 130°) + (180° - 125°) = 180°$

[Angle sum property of a triangle]

$\Rightarrow x + 50° + 55° = 180° \Rightarrow x = 75°$.

22. (d): $\angle ACB = 180° - (50° + 75°) = 55°$

[Angle sum property of a triangle]

$\therefore \angle DCE = \angle ACB = 55°$

[Vertically opposite angles]

23. (b): Let the angle be $5x$ and $4x$.

Since, these two angles are co-interior angles.

So, we have

$5x + 4x = 180°$

$\Rightarrow 9x = 180°$

$\Rightarrow x = 20°$.

Hence, greater angle $= 5x = 5 \times 20° = 100°$

24. (d): Since, AOB is a straight line

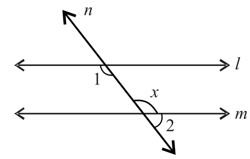
$\therefore (2x + 40°) + (x - 20°) + x = 180°$

$\Rightarrow 4x + 20° = 180°$

$\Rightarrow 4x = 160°$

$\Rightarrow x = 40°$

25. (b): Since $l \parallel m$ and n is a transversal line.



$\therefore \angle x = \angle 1$ (Alternate interior angles)

and $\angle x + \angle 2 = 180°$ (Linear pair)

$\Rightarrow \angle 1 + \angle 2 = 180°$

26. (a) : x and y forms a complementary pair

$\Rightarrow x + y = 90°$

Now, $x + 47° + y + 43° = x + y + 47° + 43°$

$= x + y + 90° = 90° + 90° = 180°$

$\therefore (x + 47°)$ and $(y + 43°)$ form a supplementary pair.

27. (c) : Let A, B, C be the three angles of a triangle.

$\angle A = \angle B + \angle C$ [Given]

Also, $\angle A + \angle B + \angle C = 180°$

[Angle sum property of a triangle]

$\Rightarrow \angle A + \angle A = 180°$

$\Rightarrow 2\angle A = 180° \Rightarrow \angle A = 90°$

$\therefore$ Triangle is a right angled triangle.

28. (a) : Given that $AB \parallel CD$ and $CD \parallel EF$

$\therefore AB \parallel CD \parallel EF$

$x + y = 180°$ ...(i) [Co-interior Angles]

$y + z = 180°$ ...(ii)

[$\angle DHI = y$, (Vertically Opposite Angles)]

Given that $y : z = 4 : 5$

$\Rightarrow y = \frac{4}{5}z$

Substituting in (ii), we get

$\frac{4z}{5} + z = 180° \Rightarrow \frac{9z}{5} = 180°$

$\Rightarrow z = \frac{180° \times 5}{9} \Rightarrow z = 100°$

Substituting value of z in (ii), we get

$y + 100° = 180° \Rightarrow y = 80°$

From (i), we have

$x + 80° = 180°$

$\Rightarrow x = 100°$.

29. **(a) :** AB and CD intersect at O.

$\Rightarrow \angle AOC = \angle BOD$ (vertically opposite angles)

$\Rightarrow \angle AOC = 60°$

Now, $\angle AOC + \angle BOE = 100°$(given)

$\Rightarrow \angle BOE = 100° - 60°$

$\Rightarrow \angle BOE = 40°$

AOB is a straight line.

$\Rightarrow \angle AOC + \angle COE + \angle EOB = 180°$

$\Rightarrow 60° + \angle COE + 40° = 180°$

$\Rightarrow \angle COE = 180° - 100°$

$\Rightarrow \angle COE = 80°$

Reflex $\angle COE = 360° - 80° = 280°$

30. **(c) :** $x : y = 3 : 2$

Let $x = 3a$ and $y = 2a$

Now, XOY is a straight line.

$\therefore x + y + \angle POY = 180°$

$\Rightarrow 3a + 2a + 70° = 180°$

$\Rightarrow 5a = 110°$

$\Rightarrow a = 22°$

Hence, $x = 3a = 3 \times 22° = 66°$

and $y = 2a = 2 \times 22° = 44°$

Now, XY and MN intersect at O.

$\therefore z = x + \angle POY$ (vertically opposite angles)

$\Rightarrow z = 66° + 70°$

$\Rightarrow z = 136°$

Fill in the Blanks

1. **180°**
2. **Parallel**
3. **70° :** $CD \parallel AB$ and $\angle AGE = 110°$

$\Rightarrow \angle BGH = \angle AGE = 110°$ [Vertically Opposite Angles]

And, $\angle BGH + \angle GHD = 180°$ [Co-interior Angles]

$\Rightarrow \angle GHD = 180° - 110° = 70°$

4. **95° :** $AB \parallel CD$

$\Rightarrow \angle 1 = \angle BLM$ [Vertically Opposite Angles]

$\Rightarrow \angle BLM = 95°$

$\therefore \angle 2 = 95°$ [Corresponding Angles]

5. **43 :** AOB is straight line

$\Rightarrow (x)° + (x + 3)° + (2x + 5)° = 180°$

$\Rightarrow 4x = 180 - 8$

$\Rightarrow 4x = 172 \Rightarrow x = 43$

6. **120° :** $\angle BOC = \angle COA + 60°$

BOA is a straight line

$\therefore \angle BOC + \angle COA = 180°$

$\Rightarrow 2\angle COA = 180° - 60°$

$\Rightarrow \angle COA = 60°$

So, $\angle BOC = 120°$

7. **16° :** For BOA to be a straight line

$\angle BOC + \angle COA = 180°$

$\Rightarrow 7x + 20° + 3x = 180°$

$\Rightarrow 10x = 160° \quad \therefore \quad x = 16°$

8. $\mathbf{\frac{80}{3}}$ **:** AOB is a straight line.

$\Rightarrow \angle AOC + \angle COD + \angle DOB = 180°$

$\Rightarrow (x + 40)° + (x + 20)° + (x + 40)° = 180°$

$\Rightarrow 3x + 100 = 180 \quad \Rightarrow \quad 3x = 80$

$\therefore \quad x = \frac{80}{3}$

9. **59° :** $2x + 4 + x - 1 = 180°$ [Linear Pair]

$\Rightarrow 3x + 3 = 180°$

$\Rightarrow 3x = 177° \Rightarrow x = 59°$

10. **55°, 125°, 125° :**

$\angle x = 55°$ [Vertically Opposite Angles]

Also, $\angle x + \angle y = 180°$ [Linear Pair]

$\Rightarrow \angle y = 180° - 55° = 125°$

Now, $\angle z = \angle y = 125°$ [Vertically Opposite Angles]

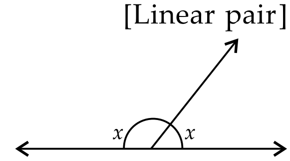
$\therefore \angle x = 55°, \angle y = 125°, \angle z = 125°$.

True or False

1. **True**
2. **False :** The measure of each adjacent angle that are equal may not be equal to 90°.
3. **False :** Angles forming a linear pair cannot be both acute angles because then their sum cannot be 180°.
4. **True :** $x + x = 180°$ [Linear pair]

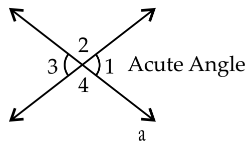
$\Rightarrow 2x = 180°$

$\Rightarrow x = 90°$



$\therefore$ If each angle forming a linear pair are equal then each angle is of measure 90°.

5. **True :** $\angle 1 + \angle 2 = 180°$ [Linear pair]



If $\angle 1 < 90°$ (acute)

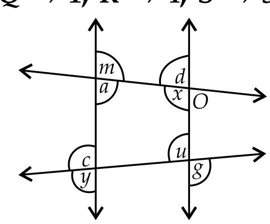
$\Rightarrow$ $\angle 2 > 90°$ (obtuse)

$\Rightarrow$ $\angle 1$ and $\angle 3$ are acute and $\angle 2$ and $\angle 4$ are obtuse.

6. **False :** If two parallel lines are intersected by a transversal, then corresponding angles are equal.
7. **True**
8. **False :** Two lines must be parallel.
9. **True**
10. **False :** Sum of supplementary angles is 180°.

Match the Following

1. **(b): P $\rightarrow$ 2; Q $\rightarrow$ 1; R $\rightarrow$ 4; S $\rightarrow$ 3**



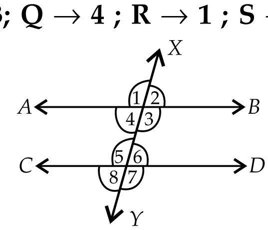
(P) Angles m and y $\rightarrow$ Alternate exterior pair of angles

(Q) Angles a and d $\rightarrow$ Alternate interior pair of angles

(R) Angles d and u $\rightarrow$ Corresponding angles

(S) Angles u and g $\rightarrow$ Vertically opposite angles.

2. **(d): P $\rightarrow$ 3; Q $\rightarrow$ 4 ; R $\rightarrow$ 1 ; S $\rightarrow$ 2**



(P) Corresponding angles $\rightarrow \angle 1 = \angle 5$

(Q) Alternate interior angles $\rightarrow \angle 4 = \angle 6$

(R) Altenate exterior angles $\rightarrow \angle 1 = \angle 7$

(S) Co-interior angles $\rightarrow \angle 4 + \angle 5 = 180°$

Assertion & Reason Type

1. **(d):** Two adjacent angles do not always form a linear pair.

In a linear pair of angles two non-common arms are opposite rays.

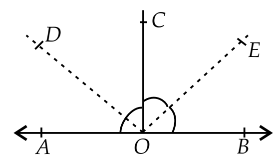
Assertion: False; **Reason:** True

2. **(b):** $\angle AOC + \angle BOC = 180°$ [Linear Pair]

$$\frac{1}{2}(\angle AOC + \angle BOC) = \frac{180°}{2}$$

$$\frac{1}{2}\angle AOC + \frac{1}{2}\angle BOC = 90°$$

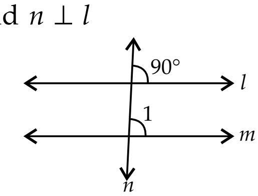
$\Rightarrow$ $\angle DOC + \angle EOC = 90°$.



$\therefore$ The bisectors of the angles of a linear pair are at right angles.

Assertion: True; **Reason:** True but is not the correct explanation of assertion.

3. **(c) :** $l \parallel m$ and $n \perp l$



$\Rightarrow$ $\angle 1 = 90°$ [Corresponding angles]

$\Rightarrow$ $n \perp m$

Reason is false. It can be stated in case of parallel lines.

Assertion: True; **Reason :** False.

4. **(d):** $XY \parallel PQ$ and EF is a transversal

$\Rightarrow$ $\angle EBQ = \angle EAY$ (Corresponding angles)

$\Rightarrow$ $20° + y = 60°$

$\Rightarrow$ $y = 60° - 20°$

$\Rightarrow$ $y = 40°$

Now, in ΔABD,

$x + 20 = \angle EAD$ (External angle property)

$\Rightarrow$ $x + 20° = 60° + 35$

$\Rightarrow$ $x = 95° - 20°$

$\Rightarrow$ $x = 75°$

Assertion: False; **Reason:** True

5. **(c) :** In ΔAFE, by pythagoras theorem.

$\angle F + \angle A + \angle AEF = 180°$

$\Rightarrow 30° + 90° + \angle AEF = 180°$

$\Rightarrow \angle AEF = 180° - 120°$

$\Rightarrow \angle AEF = 60°$

AB and FC intersect at E

$\therefore \angle AEF = \angle BEC$ (vertically opposite angles)

$\Rightarrow \angle BEC = 60°$

Now, $AB \parallel CD$ and EC is a transversal

$\Rightarrow \angle BEC + \angle ECD = 180°$ (Co-interior angles)

$\Rightarrow \angle ECD = 180° - 60°$

$\Rightarrow \angle ECD = 120°$

Assertion: True; **Reason:** False.

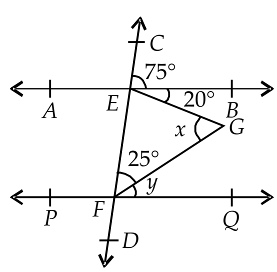
Comprehension Type

PASSAGE-I

1. **(b):** $\angle CEB = \angle EFQ$ [Corresponding angles]

$\Rightarrow 25° + y = 75°$

$\Rightarrow y = 50°$



Also, $\angle FEB + \angle EFQ = 180°$ [Co-interior angles]

$(\angle FEG + \angle GEB) + (25° + y°) = 180°$

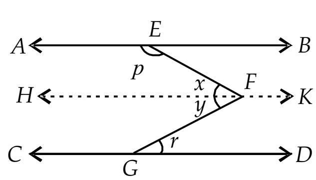
$\angle FEG + 20° + 25° + 50° = 180°$

$\Rightarrow \angle FEG = 85°$

In ΔEFG, $\angle FEG + \angle EFG + x = 180°$

$\Rightarrow 85° + 25° + x = 180° \Rightarrow x = 70°$.

2. **(b):** Draw HK parallel to AB, let $\angle EFH$ be x and



$\angle HFG$ be y such that $x + y = q$.

$p + x = 180°$ [Co-interior Angles]

$y = r$ [Alternate Interior Angles]

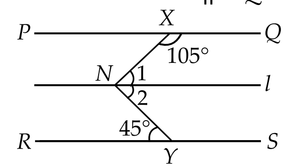
Adding both equations, we get

$p + x + y = 180° + r$

$\Rightarrow p + q = 180° + r$

$\Rightarrow p + q - r = 180°$

3. **(d):** Construction : Draw $l \parallel PQ$



$\Rightarrow \angle 1 + \angle QXN = 180°$ [Co-interior Angles]

$\therefore \angle 1 = 180° - 105° = 75°$

And,

$\therefore \angle 2 = \angle RYN$ [Alternate Interior Angles]

$\Rightarrow \angle 2 = 45°$

$\Rightarrow \angle XNY = \angle 1 + \angle 2 = 75° + 45° = 120°$

PASSAGE-II

1. **(d):** $y + 2y + 3y = 180°$ [Linear Pair]

$\Rightarrow 6y = 180° \Rightarrow y = 30°$

$\therefore$ largest angle $= 3y = 3 \times 30° = 90°$

2. **(c) :** $\angle ROA = x$, $\angle POA = 4x$ and

$= \angle ROQ = 45°$

POQ is a straight line

$\Rightarrow \angle ROA + \angle AOP + \angle ROQ = 180°$

[Linear Pair]

$\Rightarrow x + 4x + 45° = 180°$

$\Rightarrow 5x = 180° - 45° = 135°$

$\Rightarrow x = 27°$

3. **(a) :** $\angle AOC + \angle BOC = 180°$ [Linear Pair]

Let $\angle AOC = 2x$ and $\angle BOC = 3x$

$\therefore 2x + 3x = 180° \Rightarrow x = 36°$

Thus, $\angle AOC = 72°$, $\angle BOC = 108°$,

$\angle AOD = 108°$, $\angle BOD = 72°$.

PASSAGE-III

1. **(c) :** $\angle 1 = 120°$ [Vertically Opposite Angles]

For $l \parallel m$, co-interior angles must be supplementary, so $\angle 1 + \angle 2 = 180°$

$\Rightarrow 120° + \angle 2 = 180° \quad \therefore \quad \angle 2 = 60°$

2. **(c) :** Since ABC is an isosceles triangle.

$\therefore AB = AC \Rightarrow \angle B = \angle C$

Now, in ΔABC,

$\angle A + \angle B + \angle C = 180°$

[Angle sum property of Δ]

$\Rightarrow 2\angle B = 180° - 50°$

$\Rightarrow 2\angle B = 130° \Rightarrow \angle B = 65°$

Since $\angle PAC = \angle ACB$ [Alternate Interior Angles]

$\therefore \angle PAC = 65°$

$\angle PAC + \angle DAP + \angle BAC = 180°$ [Linear Pair]

$\therefore \angle DAP = 180° - 65° - 50°$

$\Rightarrow \angle ADP = 65°$.

3. **(d):** $AB \parallel DE$ [Given]
$\angle ABC = \angle EDC$ [Alternate Interior Angles]
$\Rightarrow \angle EDC = 45°$
Now, in ΔDEC,
$\angle DCA$ is exterior angle
So, $\angle DCA = \angle DEC + \angle EDC$
[Exterior angle property of a Δ]
$= 75° + 45° = 120°$

Subjective Problems

Very Short Answer Type

1. $x + 25° = 180°$ [Co-interior Angles]
$\therefore x = 180° - 25° = 155°$
Now $y + 25° = 75°$ [Alternate Interior Angles]
$\therefore y = 75° - 25° = 50°$.
Hence, $x = 155°$, $y = 50°$.

2. $\angle EGB = \angle FED$ [Corresponding Angles]
$\therefore y + 20° = 58°$
$\Rightarrow y = 58° - 20° \Rightarrow y = 38°$
Again, $y + x = \angle FEO$
[Exterior angle of ΔEOG]
$\therefore 38° + x = 58° + 22°$
$\Rightarrow x = 80° - 38° \Rightarrow x = 42°$
Hence, $x = 42°$, $y = 38°$

3. $\angle DEC = 55°$ and $\angle ACB = b$
$\Rightarrow b = 55°$ [Corresponding Angles]
In ΔABC, $\angle B = 180° - (60° + b°)$
[Angle sum property of a triangle]
$\angle B = 180° - (60° + 55°)$
$\angle B = 180° - 115° = 65°$
$\therefore a = \angle B = 65°$ [Corresponding Angles]

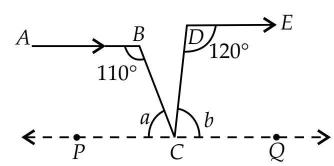
4. In ΔABC, $\angle BAC = 180° - (50° + 30°) = 100°$
[Angle sum property of a triangle]
In ΔBAD, $\angle BAD = 180° - (90° + 50°) = 40°$
and $\angle BAE = \frac{1}{2}\angle BAC = \frac{100°}{2} = 50°$
Now, $\angle DAE = x = \angle BAE - \angle BAD$
$\Rightarrow x = 50° - 40° = 10°$
Hence, $x = 10°$

5. In ΔTQR, we have
$\angle TQR + \angle QRT + \angle RTQ = 180°$
[Angle sum property of Δ]
$\Rightarrow 40° + 90° + x = 180°$
$\Rightarrow 130° + x = 180° \Rightarrow x = 50°$

6. $AB \parallel EF$, $\angle ABC = 70°$, $\angle EFD = 40°$
Since $\angle ABC = \angle CEF$ [Alternate interior angles]
$\Rightarrow \angle CEF = 70°$
Also from ΔDEF
$x = \angle E + \angle F$ [Exterior angle property of a Δ]
$\Rightarrow x = 70° + 40° = 110°$

7. $\because \angle POR$ and $\angle QOR$ form a linear pair
$\therefore \angle POR + \angle QOR = 180°$
or $a + b = 180°$ ...(1)
But $a - b = 80°$ ...(2) [Given]
Adding eq. (1) and (2), we get
$2a = 260° \quad \therefore a = \frac{260°}{2} = 130°$
Substituting the value of a in (1), we get
$130° + b = 180°$
$\Rightarrow b = 180° - 130° = 50°$
$\therefore a = 130°, b = 50°$

8. Draw PCQ parallel to AB and DE.
$a + 100° = 180°$ [Co-interior angles]
$\Rightarrow a = 80°$
Now, $\angle b + 120° = 180°$ [Co-interior Angles]
$\Rightarrow b = 60°$



Since, PCQ is a straight line
$\Rightarrow \angle a + \angle BCD + \angle b = 180°$
$\Rightarrow 80° + \angle BCD + 60° = 180°$
$\Rightarrow \angle BCD = 40°$.

9. Here $x = \angle EDC = 70°$
[Corresponding angles as $AB \parallel DC$]
and $\angle CBA = 90°$
Now, $\angle ADB = x = 70°$ [$AD = DB$]
In ΔABD, $\angle ABD = 180° - x - x$
$= 180° - 70° - 70° = 40°$
And $\angle BDC = \angle ABD = 40°$ [Alternate angles]
$\Rightarrow y = 40°$
Since, $AB \parallel DC$
$\Rightarrow z + \angle CBA = 180°$ [Co-interior angles]
$\Rightarrow z = 180° - 90° = 90° \Rightarrow z = 90°$.

10. $PF \parallel QE \Rightarrow x = 70°$ [Corresponding angles]
Also $\angle APS = \angle BPQ = 100°$
[Vertically opposite angles]
As $AB \parallel CD$
$\Rightarrow \angle BPQ + \angle PQD = 180°$ [Co-interior angles]
$\Rightarrow 100° + 70° + y = 180°$
$\Rightarrow y = 180° - 170° = 10°$
$\therefore x = 70°$ and $y = 10°$

Short Answer Type

1. $\angle BAC = (180° - 108°) = 72°$

Divide 72 in the ratio 1 : 3 we get 18 and 54.

$\therefore \angle BAD = 18°$ and $\angle DAC = 54°$

Also, $AD = DB \Rightarrow \angle DBA = \angle BAD = 18°$, etc.

Now, in ΔABC

$\angle CBA + \angle BCA = \angle EAC \Rightarrow 18 + x = 108°$

$\Rightarrow x = 90°$

2. $y + 55° = 180°$ [Co-interior Angles]

$\Rightarrow y = 180° - 55° = 125°$

Again $x = y$

[$AB \parallel CD$, Corresponding Angles]

Therefore $x = 125°$

Now, since $AB \parallel CD$ and $CD \parallel EF$, therefore, $AB \parallel EF$.

So, $\angle EAB + FEA = 180°$ [Co-interior Angle]

Therefore, $90° + z + 55° = 180°$

$\Rightarrow z = 35°$.

3. $\angle 1 = 90°$ [Co-interior Angles, $PQ \parallel RS$]

$\angle 3 = 36°$ [Vertically Opposite Angles]

$\angle 2 + \angle 3 = 90°$

[$\angle 1$, $\angle 2$ and $\angle 3$ from a linear pair]

$\Rightarrow \angle 2 = 90° - 36° \Rightarrow \angle 2 = 54°$

$\angle 7 = 90°$ [Linear Pair]

$\angle 4 + \angle 7 = \angle 1 + 36°$

[Alternate Interior Angles]

$\Rightarrow \angle 4 = 36°$

$\angle 5 = 180° - 36°$ [Linear Pair]

$\Rightarrow \angle 5 = 144°$

$\angle 6 = \angle 4 = 36°$ [Vertically Opposite Angles]

Hence, $\angle 1 = 90° = \angle 7$, $\angle 2 = 54°$,

$\angle 3 = \angle 4 = \angle 6 = 36°$, $\angle 5 = 144°$.

4. Since, $AB \parallel DC$

$\therefore a = x$ ...(1) [Alternate interior angles]

and $b = y$ ...(2) [Alternate interior angles]

Adding (1) and (2), we get

$a + b = x + y$.

5. $\angle CDE = \angle BAC = 65°$ ...(i)

[Alternate Interior Angles]

$y = 180° - 125°$ [Linear Pair]

$\therefore y = 55°$

$\angle ACB = y = 55°$ [Corresponding Angles]

In ΔACB, we have

$\angle B + \angle A + \angle ACB = 180°$

[Angle sum property of a triangle]

$\therefore \angle B = 180° - (65° + 55°)$

$\therefore \angle B = 60°$

and $x = \angle ABC = 60°$ [Corresponding Angles]

Hence, $x = 60°$, $y = 55°$.

6. $PQRST$ is a regular pentagon

$\therefore \angle P = \angle Q = \angle R = \angle S = \angle T = \dfrac{540°}{5} = 108°$ ..(i)

PL is the bisector of $\angle TPQ$

$\therefore \angle QPL = \dfrac{1}{2} \times 108° = 54°$ ...(ii)

In quadrilateral $PQRL$,

$\angle QPL + \angle PQR + \angle QRL + \angle RLP = 360°$

[Angle sum property of a quadrilateral]

$\Rightarrow 54° + 108° + 108° + \angle RLP = 360°$

$\Rightarrow \angle RLP = 360° - 270° = 90°$

In ΔLMR, $\angle MRL + \angle RLM + \angle RML = 180°$

[Angle sum property of a triangle]

$\Rightarrow 54° + 90° + \angle RML = 180°$

[MR is the bisector of $\angle SRQ$]

$\Rightarrow \angle RML = 180° - 144° = 36°$.

7. $AB \parallel DE$; $DE \parallel FG$; $CD \parallel EF$

$\angle 2 = 55°$, $\angle 4 = 60°$

Now, $EF \parallel CD$

$\Rightarrow \angle 4 = \angle 3$ [Alternate Interior Angles]

$\Rightarrow \angle 3 = 60°$

and $DE \parallel FG$

$\Rightarrow \angle 1 + \angle 4 = 180°$ [Co-interior Angles]

$\Rightarrow \angle 1 = 180° - 60° = 120°$

$\Rightarrow \angle 1 = 120°$, $\angle 3 = 60°$.

8. **Given :** Ray OE bisects $\angle AOC$,

i.e., $\angle 1 = \angle 2$,

Ray OF bisects $\angle COB$.

i.e., $\angle 3 = \angle 4$.

Also $OE \perp OF$

To Prove : Points A, O and B are collinear.

Proof : $OE \perp OF$

$\Rightarrow \angle EOF = 90° \Rightarrow \angle 2 + \angle 3 = 90°$

Multiplying both sides by 2

$\Rightarrow 2\angle 2 + 2\angle 3 = 180°$

$\Rightarrow (\angle 2 + \angle 2) + (\angle 3 + \angle 3) = 180°$

$\Rightarrow (\angle 1 + \angle 2) + (\angle 3 + \angle 4) = 180°$

[$\angle 2 = \angle 1$, $\angle 4 = \angle 3$]

$\Rightarrow \angle AOC + \angle BOC = 180°$

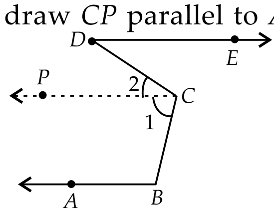
By the converse of Linear Pair Axiom

Ray OA and Ray OB are opposite rays.

$\Rightarrow AOB$ is a line.

Hence, the points A, O and B are collinear.

9. Through C, draw CP parallel to AB and DE.



Now $\angle ABC + \angle 1 = 180°$ ...(1)

[Co-interior angles]

Also, $\angle 2 = \angle CDE$ ...(2)

[Alternate interior angles]

Adding (1) and (2), we get

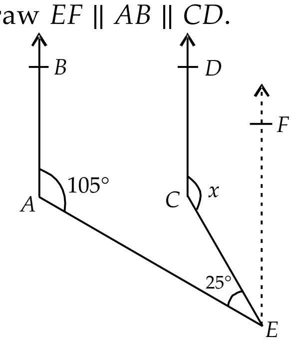
$\therefore$ $\angle ABC + \angle 1 + \angle 2 = 180° + \angle CDE$

$\Rightarrow$ $\angle ABC + \angle BCD = 180° + \angle CDE$

[$\because$ $\angle 1 + \angle 2 = \angle BCD$]

Hence proved.

10. From E, draw $EF \parallel AB \parallel CD$.



Now, $EF \parallel CD$ and CE is the transversal.

$\therefore$ $\angle DCE + \angle CEF = 180°$ [Co-interior angles]

$\Rightarrow$ $x + \angle CEF = 180°$

$\Rightarrow$ $\angle CEF = (180° - x)$

Again, $EF \parallel AB$ and AE is the transversal.

$\therefore$ $\angle BAE + \angle AEF = 180°$ [Co-interior angles]

$\Rightarrow$ $105° + 25° + (180° - x) = 180° \Rightarrow x = 130°$.

Hence, $x = 130°$.

Long Answer Type

1. In ΔPQR, $\angle Q = 60°$, $\angle R = 45°$

$\therefore$ $\angle QPR = 180° - (\angle Q + \angle R)$

$= 180° - (60° + 45°)$

$= 180° - 105° = 75°$

As PS is angle bisector of $\angle QPR$

$$\therefore \angle QPS = \frac{\angle QPR}{2} = \frac{75°}{2} = 37.5°$$

Also, ΔPTQ is a right angled triangle.

So, $\angle QPT = 180° - (90° + 60°)$

$= 180° - 150° = 30°$

$\therefore$ $\angle TPS = \angle QPS - \angle QPT$

$= 37.5° - 30° = 7.5°$.

2. Here, we have $2b - a = 65°$ ...(1)

and $\angle BOC = 90°$

So, $a + 90° + 2a + b + 15° + 2b = 360°$

[Complete angle]

or, $3a + 3b + 105° = 360°$

or, $3a + 3b = 360° - 105°$

or, $3a + 3b = 255°$

or, $a + b = 85°$ ...(2)

Adding (1) and (2), we get

$3b = 150° \Rightarrow b = 50°$

From (1), we have

$100° - a = 65° \Rightarrow a = 100° - 65° = 35°$

So, $\angle AOB = a = 35°$

$\angle AOD = 2b = 2 \times 50° = 100°$

and $\angle COD = 2a + b + 15°$

$= 2 \times 35° + 50° + 15°$

$= 70° + 50° + 15° = 135°$.

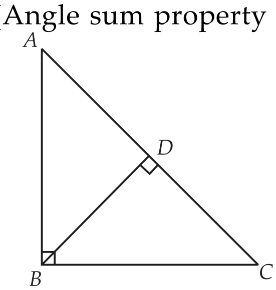
3. **Given :** ΔABC with $\angle ABC = 90°$ and $BD \perp AC$.

To Prove : $\angle ABD = \angle ACB$

Proof : In ΔABC,

$\angle A + \angle ABC + \angle C = 180°$

[Angle sum property of a triangle]



$\Rightarrow$ $\angle A + \angle C = 90°$ ...(1) [$\because$ $\angle ABC = 90°$]

Also, in ΔABD, $BD \perp AD$

So, $\angle ADB = 90°$

and $\angle A + \angle ABD + \angle ADB = 180°$

[Angle sum property of a triangle]

$\Rightarrow$ $\angle A + \angle ABD = 90°$ ...(2) [$\because$ $\angle ADB = 90°$]

From (1) and (2), we get

$\angle A + \angle C = \angle A + \angle ABD$

$\Rightarrow$ $\angle ACB = \angle ABD$.

4. In ΔABC, $\angle A + x + y = 180°$ ... (1)

[Angle sum property of a triangle]

Also, $62° + y = \angle DBC$

[Exterior angle property of a triangle]

$$\Rightarrow \frac{1}{2}(62° + y) = \frac{1}{2}\angle DBC$$

$$\Rightarrow 31° + \frac{y}{2} = \angle PBC \quad ...(2)$$

$$\text{Similarly, } 31° + \frac{x}{2} = \angle PCB \quad ... (3)$$

In ΔPBC, $\angle BPC + \angle PBC + \angle PCB = 180°$

[Angle sum property of a triangle]

$$\Rightarrow \angle BPC + 31° + \frac{y}{2} + 31° + \frac{x}{2} = 180°$$

[Using (2) and (3)]

$$\Rightarrow \angle BPC + 62° + \frac{x}{2} + \frac{y}{2} = 180°$$

$$\Rightarrow \angle BPC + 62° + \left(\frac{180° - 62°}{2}\right) = 180°$$

[using angle sum property of ΔABC]

$\angle BPC = 180° - 62° - 90° + 31° = 90° - 31° = 59°$

$\therefore \angle BPC = 59°$

5. $\angle 6 + 62° = 180°$ [Linear Pair]

$\therefore \angle 6 = 180° - 62° \Rightarrow \angle 6 = 118°$

$\angle 6 + \angle 5 = 180°$ [Co-interior Angles]

$\Rightarrow \angle 5 = 180° - 118° = 62°$

$\angle 1 = 28°$ [Alternate Interior Angle]

$\angle 4 + 28° = \angle 1 + \angle 6$

[Alternate angles as $AD \parallel BC$]

$\Rightarrow \angle 4 + 28° = 28° + 118°$

$\therefore \angle 4 = 118°$

$\angle 3 + \angle 4 = 180°$ [Linear Pair]

$\Rightarrow \angle 3 = 180° - 118° = 62°$

$\angle 2 + 28° = 180°$ [Linear Pair]

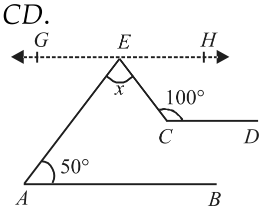
$\Rightarrow \angle 2 = 180° - 28°$

$\Rightarrow \angle 2 = 152°$

Hence, $\angle 1 = 28°$, $\angle 2 = 152°$, $\angle 3 = 62°$, $\angle 4 = 118°$, $\angle 5 = 62°$, $\angle 6 = 118°$.

Integer Answer Type

1. **(2) :** Construction : Through E draw a line $GEH \parallel AB \parallel CD$.



Now, $GE \parallel AB$ and EA is a transversal.

$\therefore \angle GEA = \angle EAB = 50°$

[Alternate interior angles]

Again, $EH \parallel CD$ and EC is a transversal.

$\therefore \angle HEC + \angle ECD = 180°$[Co-interior angles]

$\Rightarrow \angle HEC + 100° = 180° \Rightarrow \angle HEC = 80°$

Now, GEH is a straight line.

$\therefore \angle GEA + \angle AEC + \angle HEC = 180°$

$\Rightarrow 50° + x + 80° = 180°$

$\Rightarrow x = 50°$.

So, $\frac{x}{25°} = \frac{50°}{25°} = 2$.

2. **(5) :** $x = 180° - x \Rightarrow x = 90°$

$\therefore \frac{x - 60°}{6°} = \frac{90° - 60°}{6°} = 5$

3. **(3) :** Let angles be x, $2x$ and $3x$.

So, $x + 2x + 3x = 180°$

[Angle sum property of triangle]

$\Rightarrow x = 30°$

Smallest angle = 30°; Greatest angle = 90°

So, the greatest angle is 3 times to the smallest angle.

4. **(3) :** Let angles be x and $4x$.

$x + 4x = 180° \Rightarrow 5x = 180° \Rightarrow x = 36°$.

Angles are 36° and 144°

$\therefore \frac{\text{Difference of angles}}{\text{Smaller angle}} = \frac{144° - 36°}{36°} = 3.$

5. **(9) :** $a + 30° + 285° = 360°$ [Complete angle]

$\Rightarrow a = 45°$

$\Rightarrow \frac{a}{5°} = \frac{45°}{5°} = 9$

6. **(5) :** $b = 180° - (30° + a)$ [As $l \parallel m$, co-interior angles]

$= 180° - (30° + 45°)$

$\Rightarrow b = 105°$

$\Rightarrow \frac{b}{21°} = \frac{105°}{21°} = 5$

7. **(9) :** Since a and c are alternate interior angles $c = 45°$. [As $p \parallel q$]

$\therefore$ Sum of digits = 4 + 5 = 9

8. **(3) :** $c + b + d = 180°$ [Linear pair]

$\Rightarrow 45° + 105° + d = 180°$

$\Rightarrow d = 30°$

$\therefore$ Sum of digits of $d = 3 + 0 = 3$.

9. **(5) :** c and e are vertically opposite angles.

$\therefore e = 45°$.

So, $\frac{e + 5°}{10°} = \frac{50°}{10°} = 5$

10. **(4) :** b and f are vertically opposite angles.

$\therefore f = 105°$

So, $\frac{f - 5°}{25°} = \frac{100°}{25°} = 4$.