

CHAPTER 12



Exponents and Powers

Exponents (Index) or Powers

A number placed in a superscript position to the right of another number or variable indicate repeated multiplication.

For example : a^2 indicates $a \times a$

a^3 indicates $a \times a \times a$, etc.

For example : 3 raised to the eighth power indicates $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$ and would look like a 3 with a small 8 on the upper right hand side of the 3.

In the expression 2^5 , 2 is called base and 5 is called the exponent or power.

Powers with Negative Exponents

For any non-zero rational number ' a ' and a positive integer n , we define $a^{-n} = \frac{1}{a^n}$ i.e., a^{-n} is the reciprocal of a^n .

- 1 Find the value of $\left(\frac{2}{3}\right)^{-4/5}$.

$$\text{Soln.: } \left(\frac{2}{3}\right)^{-4/5} = \frac{1}{\left(\frac{2}{3}\right)^{4/5}} \quad \left[\because a^{-n} = \frac{1}{a^n} \right]$$

$$= \frac{1}{\left(\frac{(2)^{4/5}}{(3)^{4/5}}\right)} = 1 \times \frac{(3)^{4/5}}{(2)^{4/5}} = \left(\frac{3}{2}\right)^{4/5}$$

- 2 Find the multiplicative inverse of the following :

(i) 3^{-4}

(ii) 2^{-5}

(iii) 10^{-100}



ILLUSTRATION

$$\text{Soln.: (i) } 3^{-4} = \frac{1}{3^4} \quad \left(\because a^{-n} = \frac{1}{a^n} \right)$$

$\therefore 3^4$ is the multiplicative inverse of 3^{-4}

$$\text{(ii) } 2^{-5} = \frac{1}{2^5} \quad \left(\because a^{-n} = \frac{1}{a^n} \right)$$

$\therefore 2^5$ is the multiplicative inverse of 2^{-5}

$$\text{(iii) } 10^{-100} = \frac{1}{10^{100}} \quad \left(\because a^{-n} = \frac{1}{a^n} \right)$$

$\therefore 10^{100}$ is the multiplicative inverse of 10^{-100}

Laws of Exponents

(i) First law

$a^m \times a^n = a^{m+n}$, where a is a rational number and m, n are two different exponents.

3 Simplify : $(-5)^4 \times (-5)^3$

Soln.: We have, $(-5)^4 \times (-5)^3 = (-5)^{4+3} = (-5)^7$
[$\because a^m \times a^n = a^{m+n}$]



ILLUSTRATION

(ii) Second Law

$\frac{a^m}{a^n} = a^{m-n}$; $m > n$, where a is a rational number and m, n are two different exponents.

4 Simplify : $\frac{(-5)^4}{(-5)^2}$

Soln.: We have, $\frac{(-5)^4}{(-5)^2} = (-5)^{4-2} = (-5)^2$ ($\because \frac{a^m}{a^n} = a^{m-n}$)



ILLUSTRATION

(iii) Third Law

$(a^m)^n = a^{mn} = (a^n)^m$, where a is a rational number and m, n are two different exponents.

5 Simplify : $(3^2)^3$.

Soln.: We have, $(3^2)^3 = 3^{2 \times 3}$
 $(9)^3 = (3)^6 = 3 \times 3 \times 3 \times 3 \times 3 \times 3$
= 729 [$\therefore (a^m)^n = a^{mn} = (a^n)^m$]



ILLUSTRATION

(iv) Fourth Law

$(ab)^n = a^n b^n$, where a and b are two different rationals and n is exponent.

6 Simplify : $(3 \times 2)^4$.

Soln.: $(3 \times 2)^4 = 3^4 \times 2^4 = (3 \times 3 \times 3 \times 3) \times (2 \times 2 \times 2 \times 2)$
= $81 \times 16 = 1296$ [$\therefore (ab)^n = a^n b^n$]



ILLUSTRATION

(v) Fifth Law

$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$, where a and b are two different rationals and n is exponent.

7 Simplify : $\left(\frac{3}{2}\right)^4$.

Soln.: $\left(\frac{3}{2}\right)^4 = \frac{3^4}{2^4} = \frac{81}{16}$

$$\left(\because \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \right)$$



ILLUSTRATION

(vi) Sixth Law

$a^1 = a$ and $a^0 = 1$, where a is a rational number.

8 Simplify : (i) $(32)^1$ (ii) $(219)^0$
(iii) $(1000)^1$

Soln.: (i) $32^1 = 32$ (ii) $(219)^0 = 1$ (iii) $(1000)^1 = 1000$



ILLUSTRATION

Decimal Number System

Decimal Fractions : Fractions in which denominators are powers of 10 known as decimal fractions.

For example : $\frac{1}{10} = 1$ tenth $= 0.1$, $\frac{1}{100} = 1$ hundredth $= 0.01$

Expanded form of Decimal Number System

Any decimal number can be written in expanded form by using integral exponents of 10.

For example : 5743.53 in the expanded form can be expressed as follows :

$$\begin{aligned} 5743.53 &= 5 \times 1000 + 7 \times 100 + 4 \times 10 + 3 \times 1 + 5 \times \frac{1}{10} + 3 \times \frac{1}{100} \\ &= 5 \times 10^3 + 7 \times 10^2 + 4 \times 10^1 + 3 \times 10^0 + 5 \times 10^{-1} + 3 \times 10^{-2} \end{aligned}$$

Use of Exponents to Express Small Numbers in Standard Form

There are three steps :

Step I : Obtain the number and see whether the number is between 1 and 10 or it is less than 1.

Step II : If the number is between 1 and 10, then write it as the product of the number itself and 10^0 .

Step III : If the number is less than one, then move the decimal point to the right so that there is just one digit on the left side of the decimal point. Write the given number as the product of the number so obtained and 10^{-n} , where n is the number of places the decimal point has been moved to the right. The number so obtained is the standard form of the given number.



ILLUSTRATION

9 Express each of the following as a rational number of the form $\frac{p}{q}$:

(i) $(-2)^{-5}$

(ii) $\left(\frac{-2}{5}\right)^{-4}$

Soln.: We know that, if a is a non-zero rational number and n is a positive integer, then $a^{-n} = \frac{1}{a^n}$.
Thus, we have

$$(i) (-2)^{-5} = \frac{1}{(-2)^5} = -\frac{1}{32} \quad \left[\because a^{-n} = \frac{1}{a^n} \right]$$

$$(ii) \left(\frac{-2}{5}\right)^{-4} = \frac{1}{\left(\frac{-2}{5}\right)^4} \quad \left[\because a^{-n} = \frac{1}{a^n} \right]$$

$$= \frac{1}{\frac{(-2)^4}{5^4}} = \frac{1}{\frac{16}{625}} = \frac{625}{16} \quad \left[\because \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \text{ for } n > 0 \right]$$

10 Simplify :

(i) $(-4)^5 \times (-4)^{-10}$ (ii) $2^5 \div 2^{-6}$

$$\begin{aligned} \text{Soln.: (i)} \quad & (-4)^5 \times (-4)^{-10} \\ & = (-4)^{(5-10)} = (-4)^{-5} = \frac{1}{(-4)^5} \end{aligned}$$

$$\left(\because a^m \times a^n = a^{m+n}, a^{-m} = \frac{1}{a^m} \right)$$

(ii) $2^5 \div 2^{-6} = 2^{5-(-6)} = 2^{11} \quad (\because a^m \div a^n = a^{m-n})$

11 Express 4^{-3} as a power with the base 2.

Soln.: We have, $4 = 2 \times 2 = 2^2$

$$\text{Therefore, } (4)^{-3} = (2 \times 2)^{-3} = (2^2)^{-3} = 2^{2 \times (-3)} = 2^{-6} \\ [\because (a^m)^n = a^{mn}]$$

12 Express the following numbers in standard form :

(i) 0.000053

(ii) 4500000

Soln.: (i) $0.000053 = 5.3 \times 10^{-5}$

(ii) $4500000 = 4.50 \times 10^6$

13 Express the following numbers in usual form :

(i) 3.52×10^5

(ii) 7.54×10^{-4}

(iii) 3×10^{-5}

Soln.: (i) $3.52 \times 10^5 = 3.52 \times 100000 = 352000$

$$(ii) 7.54 \times 10^{-4} = \frac{7.54}{10^4} = \frac{7.54}{10000} = 0.000754$$

$$(iii) 3 \times 10^{-5} = \frac{3}{10^5} = \frac{3}{100000} = 0.00003$$

ESSENTIAL POINTS

for COMPETITIVE EXAMS

- If a is a non-zero rational number and n is a positive integer, then
 - (i) $a^n = a \times a \times a \times \dots \times a$ (n times)
 - (ii) $a^{-n} = \frac{1}{a^n}$
 - (iii) $a^0 = 1$
- If a, b are non-zero rational numbers and m, n are integers, then
 - (i) $a^m \times a^n = a^{m+n}$
 - (ii) $\frac{a^m}{a^n} = a^{m-n}$
 - (iii) $(a^m)^n = a^{mn} = (a^n)^m$
 - (iv) $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
 - (v) $(ab)^n = a^n \times b^n$
- If $\frac{a}{b}$ is any non-zero rational number and n is a positive integer, then
$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n = \frac{b^n}{a^n}$$
- a^{-n} is multiplicative inverse of a^n .
- When we have to add numbers in standard form, we convert them into numbers with the same exponents.
- Very small numbers can be expressed in standard form using negative exponents.

SOLVED EXAMPLES

- 1.** Express each of the following as a rational number of the form $\frac{p}{q}$.
- (i) $\left(\frac{3}{8}\right)^{-2} \times \left(\frac{4}{5}\right)^{-3}$
- (ii) $\left(\frac{-2}{7}\right)^{-4} \times \left(\frac{-7}{5}\right)^3$
- Soln.:** (i) We have,
- $$\begin{aligned} \left(\frac{3}{8}\right)^{-2} \times \left(\frac{4}{5}\right)^{-3} &= \frac{1}{\left(\frac{3}{8}\right)^2} \times \frac{1}{\left(\frac{4}{5}\right)^3} \quad \left[\because a^{-n} = \frac{1}{a^n} \right] \\ &= \frac{1}{\frac{3^2}{8^2}} \times \frac{1}{\frac{4^3}{5^3}} \quad \left[\because \left(\frac{a}{b}\right)^n = \left(\frac{a^n}{b^n}\right) \right] \\ &= \frac{1}{\frac{9}{64}} \times \frac{1}{\frac{64}{125}} = \frac{64}{9} \times \frac{125}{64} = \frac{125}{9} \end{aligned}$$
- (ii) $\left(\frac{-2}{7}\right)^{-4} \times \left(\frac{-7}{5}\right)^3$
- $$\begin{aligned} &= \frac{1}{\left(\frac{-2}{7}\right)^4} \times \left(\frac{-7}{5}\right)^3 = \frac{1}{\frac{(-2)^4}{7^4}} \times \frac{(-7)^3}{5^3} \\ &= \frac{7^4}{(-2)^4} \times \frac{(-7)^2}{5^2} = \frac{7 \times 7 \times 7 \times 7}{16} \times \frac{(-7) \times (-7)}{25} \\ &= \frac{7^6 \times (-1)^2}{16 \times 25} = \frac{7^6}{400} = \frac{117649}{400} \end{aligned}$$
- 2.** Find the absolute value of
- (i) $\left(\frac{2}{-3}\right)^4$ (ii) $\left(\frac{-2}{7}\right)^3$
- Soln.:** (i) $\left(\frac{2}{-3}\right)^4 = \left| \left(\frac{2}{-3}\right)^4 \right| = \left| \frac{2^4}{(-3)^4} \right| = \left| \frac{16}{81} \right| = \frac{16}{81}$
- (ii) $\left(\frac{-2}{7}\right)^3 = \left| \left(\frac{-2}{7}\right)^3 \right| = \left| \frac{(-2)^3}{(7)^3} \right| = \left| \frac{-8}{343} \right| = \frac{8}{343}$
- 3.** Find the reciprocal of
- (i) 3^4 (ii) $\left(\frac{2}{3}\right)^6$
- Soln.:** (i) Reciprocal of $3^4 = \frac{1}{3^4} = \frac{1}{81}$
- (ii) Reciprocal of $\left(\frac{2}{3}\right)^6 = \left(\frac{3}{2}\right)^6 = \frac{729}{64}$
- 4.** Evaluate : $\left\{ \left(\frac{-3}{2}\right)^2 \right\}^{-3}$
- Soln.:** We have, $\left\{ \left(\frac{-3}{2}\right)^2 \right\}^{-3} = \left(\frac{-3}{2}\right)^{2 \times (-3)}$
- $$\begin{aligned} &= \left(\frac{-3}{2}\right)^{-6} = \left(\frac{2}{-3}\right)^6 = \frac{2^6}{(-3)^6} = \frac{2^6}{(-1 \times 3)^6} \\ &= \frac{2^6}{(-1)^6 \times 3^6} = \frac{2^6}{3^6} = \frac{64}{729} \end{aligned}$$
- 5.** Express each of the following as a rational number of the form $\frac{p}{q}$:
- (i) $(2^{-1} + 3^{-1})^2$ (ii) $(2^{-1} - 4^{-1})^2$
- (iii) $\left\{ \left(\frac{4}{3}\right)^{-1} - \left(\frac{1}{4}\right)^{-1} \right\}^{-1}$
- Soln.:** We know that for any positive integer n and any rational number a , $a^{-n} = \frac{1}{a^n}$. Thus, we have
- (i) $(2^{-1} + 3^{-1})^2 = \left(\frac{1}{2} + \frac{1}{3} \right)^2 = \left(\frac{3+2}{6} \right)^2 = \left(\frac{5}{6} \right)^2 = \frac{5^2}{6^2} = \frac{25}{36}$
- (ii) $(2^{-1} - 4^{-1})^2 = \left(\frac{1}{2} - \frac{1}{4} \right)^2 = \left(\frac{2-1}{4} \right)^2 = \left(\frac{1}{4} \right)^2 = \frac{1^2}{4^2} = \frac{1}{16}$

$$\text{(iii)} \quad \left\{ \left(\frac{4}{3} \right)^{-1} - \left(\frac{1}{4} \right)^{-1} \right\}^{-1} = \left(\frac{1}{\frac{4}{3}} - \frac{1}{\frac{1}{4}} \right)^{-1}$$

$$= \left(\frac{3}{4} - \frac{4}{1} \right)^{-1} = \left(\frac{3-16}{4} \right)^{-1} = \left(\frac{-13}{4} \right)^{-1} = \frac{-4}{13}$$

6. Simplify :

$$\text{(i)} \quad (2^{-1} \times 5^{-1})^{-1} \div 4^{-1}$$

$$\text{(ii)} \quad (4^{-1} + 8^{-1}) \div \left(\frac{2}{3} \right)^{-1}$$

$$\text{Soln.: (i)} \quad (2^{-1} \times 5^{-1})^{-1} \div 4^{-1} = \left(\frac{1}{2} \times \frac{1}{5} \right)^{-1} \div \left(\frac{4}{1} \right)^{-1}$$

$$= \left(\frac{1}{10} \right)^{-1} \div \left(\frac{1}{4} \right) = \left(\frac{10}{1} \right) \div \left(\frac{1}{4} \right)$$

$$= \left(10 \div \frac{1}{4} \right) = (10 \times 4) = 40$$

$$\text{(ii)} \quad (4^{-1} + 8^{-1}) \div \left(\frac{2}{3} \right)^{-1} = \left(\frac{1}{4} + \frac{1}{8} \right) \div \left(\frac{3}{2} \right)$$

$$= \frac{(2+1)}{8} \div \frac{3}{2} = \left(\frac{3}{8} \div \frac{3}{2} \right) = \left(\frac{3}{8} \times \frac{2}{3} \right) = \frac{1}{4}$$

7. Prove that

$$\frac{1}{1+x^{b-a}+x^{c-a}} + \frac{1}{1+x^{a-b}+x^{c-b}} + \frac{1}{1+x^{b-c}+x^{a-c}} = 1$$

$$\text{Soln.: LHS} = \frac{1}{1+\frac{x^b}{x^a}+\frac{x^c}{x^a}} + \frac{1}{1+\frac{x^a}{x^b}+\frac{x^c}{x^b}} + \frac{1}{1+\frac{x^b}{x^c}+\frac{x^a}{x^c}}$$

$$= \frac{x^a}{x^a+x^b+x^c} + \frac{x^b}{x^b+x^a+x^c} + \frac{x^c}{x^c+x^b+x^a}$$

$$= \frac{x^a+x^b+x^c}{x^a+x^b+x^c} = 1 = \text{RHS.}$$

8. If $a^{1/x} = b^{1/y} = c^{1/z}$, $b^2 = ac$, then find the value

$$\text{of } \frac{x+z}{2y}.$$

$$\text{Soln.: } a^{1/x} = k \quad \left| \begin{array}{l} b^{1/y} = k \\ a = k^x \end{array} \right. \quad \left| \begin{array}{l} c^{1/z} = k \\ b = k^y \\ c = k^z \end{array} \right.$$

we have, $b^2 = ac \Rightarrow (k^y)^2 = k^x \times k^z$
 $\Rightarrow k^{2y} = k^x \times k^z \Rightarrow k^{2y} = k^{x+z} \Rightarrow 2y = x + z$ (Since the base are same \therefore equating the powers)

$$\Rightarrow \frac{x+z}{2y} = 1$$

9. If $\frac{2^x}{1+2^x} = \frac{1}{4}$, then find the value of $\frac{8^x}{1+8^x}$.

$$\text{Soln.: } \frac{2^x}{1+2^x} = \frac{1}{4}$$

$$\Rightarrow 4 \cdot 2^x = 1 + 2^x \quad [\text{By cross multiplication}]$$

$$\Rightarrow 4 \cdot 2^x - 2^x = 1 \quad \Rightarrow 2^x(4-1) = 1$$

$$\Rightarrow 3 \cdot 2^x = 1$$

$$\Rightarrow 2^x = \frac{1}{3} \quad [\text{Dividing by 3 on both sides}]$$

$$\therefore (2^x)^3 = \left(\frac{1}{3} \right)^3 = \frac{1}{27}$$

$$\therefore \frac{8^x}{1+8^x} = \frac{(2^3)^x}{1+(2^3)^x} = \frac{(2^x)^3}{1+(2^x)^3} = \frac{\frac{1}{27}}{1+\frac{1}{27}}$$

$$= \frac{1}{27} \times \frac{27}{28} = \frac{1}{28}$$

10. Arrange in descending order of magnitude $\sqrt[3]{2}, \sqrt[6]{3}, \sqrt[9]{4}$.

Soln.: L.C.M of 3, 6, 9 is 18.

$$\therefore \sqrt[3]{2} = \sqrt[3 \times 6]{2^6} = \sqrt[18]{64}$$

[For comparison either make base same or exponent same]

$$\sqrt[6]{3} = \sqrt[6 \times 3]{3^3} = \sqrt[18]{27}$$

$$\sqrt[9]{4} = \sqrt[9 \times 2]{4^2} = \sqrt[18]{16}$$

Since, $\sqrt[18]{64} > \sqrt[18]{27} > \sqrt[18]{16}$

$$\therefore \sqrt[3]{2} > \sqrt[6]{3} > \sqrt[9]{4}$$

11. Which is greater of the two : 2^{300} or 3^{200} ?

Soln.: For comparing two numbers of different base and different exponents, it is better that either the base or the exponent is made same for both numbers.

Hence, $2^{300} = (2^3)^{100} = 8^{100}$ (exponent is 100)
and $3^{200} = (3^2)^{100} = 9^{100}$ (exponent is 100)

From the above, it is evident that

$$9^{100} > 8^{100} \Rightarrow 3^{200} > 2^{300}$$

12. Solve for y if

$$\frac{\left(\frac{1}{9}\right)^{2y-1} \cdot (0.0081)^{1/3}}{\sqrt{243}} = \left(\frac{1}{3}\right)^{2y-5} \sqrt[3]{\frac{27^{y-1}}{10000}}.$$

Soln.: The given equation can be written as

$$\begin{aligned} \frac{(3^{-2})^{2y-1} \cdot (3^4 \times 10^{-4})^{1/3}}{3^{5/2}} &= \frac{3^{-(2y-5)} \cdot 3^{3(y-1)}}{10^{4/3}} \\ \Rightarrow \frac{3^{-4y+2+4/3-5/2}}{10^{4/3}} &= \frac{3^{-2y+5+y-1}}{10^{4/3}} \\ \Rightarrow 3^{-4y+5/6} &= 3^{-y+4} \\ \Rightarrow -4y + \frac{5}{6} &= -y + 4 \\ \Rightarrow -4y + y &= 4 - \frac{5}{6} = \frac{24-5}{6} = \frac{19}{6} \\ \Rightarrow -3y &= \frac{19}{6} \Rightarrow y = -\frac{19}{18} \end{aligned}$$

13. By what number should $\left(\frac{1}{2}\right)^{-1}$ be multiplied so that the product is $\left(\frac{-5}{4}\right)^{-1}$?

Soln.: Let the required number be x . Then,

$$\begin{aligned} x \times \left(\frac{1}{2}\right)^{-1} &= \left(\frac{-5}{4}\right)^{-1} \Rightarrow x \times \frac{2}{1} = \frac{4}{-5} \\ \Rightarrow 2x &= \frac{-4}{5} \Rightarrow x = \left(\frac{1}{2} \times \frac{-4}{5}\right) = \frac{-2}{5}. \end{aligned}$$

Hence, the required number is $\frac{-2}{5}$.

14. Find the value of m for $5^{2m} \div 5^{-1} = 5^5$.

Soln.: $5^{2m} \div 5^{-1} = 5^5 \Rightarrow 5^{2m-(-1)} = 5^5 \Rightarrow 5^{2m+1} = 5^5$

Since, both sides have the same base, therefore their exponents must be equal.

$$\begin{aligned} \text{So, } 2m+1 &= 5 \Rightarrow 2m = 5-1 \Rightarrow 2m = 4 \\ \Rightarrow m &= 2 \end{aligned}$$

15. Simplify :

$$\begin{array}{ll} (\text{i}) \quad \left(\frac{5}{8}\right)^{-7} \times \left(\frac{8}{5}\right)^{-5} & (\text{ii}) \quad \left(\frac{-2}{3}\right)^{-2} \times \left(\frac{4}{5}\right)^{-3} \\ (\text{iii}) \quad \left(\frac{3}{4}\right)^{-4} \div \left(\frac{3}{2}\right)^{-3} & (\text{iv}) \quad \left(\frac{3}{7}\right)^{-2} \times \left(\frac{7}{6}\right)^{-3} \end{array}$$

$$\begin{aligned} \text{Soln. (i)} \quad \text{We have, } \left(\frac{5}{8}\right)^{-7} \times \left(\frac{8}{5}\right)^{-5} &= \frac{5^{-7}}{8^{-7}} \times \frac{8^{-5}}{5^{-5}} \\ &= 5^{-7+5} \times 8^{-5+7} = 5^{-2} \times 8^2 = \frac{8^2}{5^2} = \frac{64}{25} \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad \text{We have, } \left(\frac{-2}{3}\right)^{-2} \times \left(\frac{4}{5}\right)^{-3} &= \frac{(-2)^{-2}}{3^{-2}} \times \frac{4^{-3}}{5^{-3}} \\ &= \frac{3^2}{(-2)^2} \times \frac{5^3}{4^3} = \frac{9}{4} \times \frac{125}{64} = \frac{1125}{256} \\ \text{(iii)} \quad \text{We have, } \left(\frac{3}{4}\right)^{-4} \div \left(\frac{3}{2}\right)^{-3} &= \left(\frac{3}{4}\right)^{-4} \times \left(\frac{3}{2}\right)^3 = \left(\frac{3}{4}\right)^{-4} \times \left(\frac{3}{2}\right)^3 = \frac{3^{-4}}{4^{-4}} \times \frac{3^3}{2^3} \\ &= \frac{3^{-4} \times 3^3}{(2^2)^{-4} \times 2^3} = \frac{3^{-4} \times 3^3}{2^{-8} \times 2^3} = \frac{3^{-4+3}}{2^{-8+3}} = \frac{3^{-1}}{2^{-5}} \\ &= \frac{2^5}{3^1} = \frac{32}{3} \end{aligned}$$

(iv) We have,

$$\begin{aligned} \left(\frac{3}{7}\right)^{-2} \times \left(\frac{7}{6}\right)^{-3} &= \frac{3^{-2}}{7^{-2}} \times \frac{7^{-3}}{6^{-3}} = \frac{3^{-2}}{7^{-2}} \times \frac{7^{-3}}{(2 \times 3)^{-3}} \\ &= \frac{3^{-2}}{7^{-2}} \times \frac{7^{-3}}{2^{-3} \times 3^{-3}} = \frac{3^{-2}}{3^{-3}} \times \frac{7^{-3}}{7^{-2}} \times \frac{1}{2^{-3}} \\ &= 3^{-2+3} \times 7^{-3+2} \times 2^3 = 3 \times 7^{-1} \times 2^3 \\ &= 3 \times \frac{1}{7} \times 8 = \frac{24}{7} \end{aligned}$$

16. Write the following numbers in standard form :

$$\begin{array}{ll} (\text{i}) \quad 0.4579 & (\text{ii}) \quad 0.000007 \\ (\text{iii}) \quad 0.0000021 & (\text{iv}) \quad 216000000 \\ (\text{v}) \quad 0.0000529 \times 10^4 & (\text{vi}) \quad 9573 \times 10^{-4} \end{array}$$

Soln. (i) To express 0.4579 in standard form, the decimal point is moved through one place only to the right so that there is just one digit on the left of the decimal point.

$\therefore 0.4579 = 4.579 \times 10^{-1}$ is in standard form.

(ii) $0.000007 = 7 \times 10^{-6}$ [∴ The decimal point is moved 6 places to the right]

(iii) $0.0000021 = 2.1 \times 10^{-6}$ [∴ The decimal point is moved 6 places to the right]

(iv) $216000000 = 2.16 \times 10^8$ [∴ The decimal point is moved 8 places to the left]

(v) $0.0000529 \times 10^4 = 5.29 \times 10^{-5} \times 10^4$
[∴ The decimal point is moved 5 places to the right]
 $= 5.29 \times 10^{-5+4} = 5.29 \times 10^{-1}$

(vi) $9573 \times 10^{-4} = 9.573 \times 10^3 \times 10^{-4}$
[∴ The decimal point is moved 3 places to the left]
 $= 9.573 \times 10^{3+(-4)} = 9.573 \times 10^{-1}$

17. Write the following numbers in usual form :

(i) 1.785×10^7 (ii) 5.1×10^{-7}

Soln.: (i) $1.785 \times 10^7 = 1.785 \times 10000000$
 $= 17850000$

(ii) $5.1 \times 10^{-7} = \frac{5.1}{10^7} = \frac{5.1}{10000000} = 0.00000051$

18. Solve $12^x = 144$ and find the value of x .

Soln.: Given, $12^x = 144$

$\Rightarrow 12^x = 12^2$

On comparing the powers we get,

$x = 2$

19. Evaluate : (i) $(13^2 - 5^2)^{\frac{3}{2}}$

(ii) $(1^3 + 2^3 + 3^3 + 4^3)^{\frac{-3}{2}}$

Soln.: (i) $(13^2 - 5^2)^{\frac{3}{2}} = [(13+5) \times (13-5)]^{\frac{3}{2}}$
 $= [18 \times 8]^{\frac{3}{2}} = [3 \times 3 \times 2 \times 2 \times 2 \times 2]^{\frac{3}{2}}$

$$= [3^2 \times 2^4]^{\frac{3}{2}} = [3^2]^{\frac{3}{2}} \times [2^4]^{\frac{3}{2}} \\ = 3^{2 \times \frac{3}{2}} \times 2^{4 \times \frac{3}{2}} = 3^3 \times 2^6 = 1728$$

(ii) $(1^3 + 2^3 + 3^3 + 4^3)^{\frac{-3}{2}}$

$$= (1+8+27+64)^{\frac{-3}{2}}$$

$$= (100)^{-3/2} = (10^2)^{-3/2} = 10^{-3} = \frac{1}{1000}$$

20. Solve each of the following exponential equations.

(i) $6^x = 216$ (ii) $6^{x-4} = 1$

Soln.: (i) $6^x = 216 \Rightarrow 6^x = 6^3 \Rightarrow x = 3$

(ii) $6^{x-4} = 6^0 \Rightarrow x-4 = 0 \Rightarrow x = 4$

NCERT SECTION

Exercise 12.1

1. Evaluate.

(i) 3^{-2} (ii) $(-4)^{-2}$ (iii) $\left(\frac{1}{2}\right)^{-5}$.

Soln.: (i) We have,

$$3^{-2} = \frac{1}{3^2} = \frac{1}{3 \times 3} = \frac{1}{9}. \quad \left[\because a^{-m} = \frac{1}{a^m} \right]$$

(ii) We have, $(-4)^{-2} = \frac{1}{(-4)^2} = \frac{1}{(-4) \times (-4)} = \frac{1}{16}$.

$$\left[\because a^{-m} = a \frac{1}{a^m} \right]$$

(iii) We have, $\left(\frac{1}{2}\right)^{-5} = \left(\frac{2}{1}\right)^5 = \frac{2 \times 2 \times 2 \times 2 \times 2}{1 \times 1 \times 1 \times 1 \times 1}$

$$= \frac{32}{1} = 32. \quad \left[\because \left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m \right]$$

2. Simplify and express the result in power notation with positive exponent.

(i) $(-4)^5 \div (-4)^8$

(ii) $\left(\frac{1}{2^3}\right)^2$

(iii) $(-3)^4 \times \left(\frac{5}{3}\right)^4$

(iv) $(3^{-7} \div 3^{-10}) \times 3^{-5}$

(v) $2^{-3} \times (-7)^{-3}$

Soln.: (i) We have, $(-4)^5 \div (-4)^8$

$$= (-4)^{5-8} = (-4)^{-3} = \frac{1}{(-4)^3}$$

$$\left[\because a^m \div a^n = a^{m-n}, \quad a^{-m} = \frac{1}{a^m} \right]$$

(ii) We have, $\left(\frac{1}{2^3}\right)^2 = \frac{1}{(2^3)^2} = \frac{1}{2^6}$

$$\left[\because (a^m)^n = a^{mn} \right]$$

(iii) We have, $(-3)^4 \times \left(\frac{5}{3}\right)^4$

$$= (-1)^4 (3)^4 \times \frac{(5)^4}{(3)^4} = \frac{3^4}{3^4} \cdot 5^4$$

$$= 3^{4-4} \cdot 5^4 = 3^0 \times 5^4 = 1 \times 5^4 = (5)^4$$

$$\left[\because \frac{a^m}{a^n} = a^{m-n} \right]$$

(iv) We have, $(3^{-7} \div 3^{-10}) \times 3^{-5}$

$$= [3^{-7} - (-10)] \times 3^{-5} \quad \left[\because a^m \div a^n = a^{m-n} \right]$$

$$= [3^{-7+10}] \times 3^{-5} = 3^3 \times 3^{-5} = 3^{3+(-5)}$$

$$\left[\because a^m \times a^n = a^{m+n} \right]$$

$$= 3^{3-5} = 3^{-2} = \frac{1}{3^2}.$$

(v) We have, $2^{-3} \times (-7)^{-3}$

$$= \frac{1}{2^3} \times \frac{1}{(-7)^3} = \frac{-1}{2^3 \cdot 7^3} = \frac{-1}{(14)^3} = \frac{1}{(-14)^3}.$$

$$[\because a^m \cdot b^m = (ab)^m]$$

3. Find the value of

(i) $(3^0 + 4^{-1}) \times 2^2$

(ii) $(2^{-1} \times 4^{-1}) \div 2^{-2}$

(iii) $\left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2}$

(iv) $(3^{-1} + 4^{-1} + 5^{-1})^0$

(v) $\left\{ \left(\frac{-2}{3} \right)^{-2} \right\}^2$

Soln.: (i) We have,

$$(3^0 + 4^{-1}) \times 2^2 = \left(1 + \frac{1}{4}\right) \times 2^2$$

$$= \left(1 + \frac{1}{4}\right) \times 4 = \left(\frac{4+1}{4}\right) \times 4 = \frac{5}{4} \times 4 = 5$$

(ii) We have,

$$\begin{aligned}(2^{-1} \times 4^{-1}) &\div 2^{-2} = [2^{-1} \times (2^2)^{-1}] \div 2^{-2} \\&= [2^{-1} \times 2^{-2}] \div 2^{-2} [\because (a^m)^n = (a^{mn})] \\&= [2^{-1+(-2)}] \div 2^{-2} [\because (a^m \times a^n) = (a^{m+n})] \\&= [2^{-1-2}] \div 2^{-2} = 2^{-3} \div 2^{-2} \\&= 2^{-3-(-2)} [\because (a^m \div a^n) = a^{m-n}] \\&= 2^{-3+2} = 2^{-1} = \frac{1}{2}\end{aligned}$$

(iii) We have, $\left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2}$

$$= (2)^2 + (3)^2 + (4)^2 = 4 + 9 + 16 = 29$$

$$\left[\because (a)^{-m} = \frac{1}{a^m}\right]$$

(iv) We have, $(3^{-1} + 4^{-1} + 5^{-1})^0$

$$= \left[\frac{1}{3} + \frac{1}{4} + \frac{1}{5}\right]^0$$

$$\left[\because (a)^{-m} = \frac{1}{a^m}\right]$$

$$= \left[\frac{20 + 15 + 12}{60}\right]^0 = \left[\frac{47}{60}\right]^0 = 1 \quad [\because a^0 = 1]$$

(v) We have,

$$\begin{aligned}\left\{\left(\frac{-2}{3}\right)^{-2}\right\}^2 &= \left\{\left(-\frac{3}{2}\right)^2\right\}^2 \left[\because \left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m\right] \\&= \left\{\left(\frac{-3}{2}\right) \times \left(\frac{-3}{2}\right)\right\}^2 = \left\{\frac{9}{4}\right\}^2 = \frac{9}{4} \times \frac{9}{4} = \frac{81}{16}\end{aligned}$$

4. Evaluate :

(i) $\frac{8^{-1} \times 5^3}{2^{-4}}$ (ii) $(5^{-1} \times 2^{-1}) \times 6^{-1}$

Soln.: (i) We have, $\frac{8^{-1} \times 5^3}{2^{-4}}$

$$= \frac{(2^3)^{-1} \times 5^3}{(2)^{-4}} = \frac{(2)^{-3} \times 5^3}{(2)^{-4}} \quad [\because (a^m)^n = a^{mn}]$$

$$= (2)^{-3-(-4)} \times 5^3 = (2)^{-3+4} \times 5^3 = (2)^1 \times 5^3$$

$$\left[\because \frac{a^m}{a^n} = a^{m-n}\right]$$

$$= 2 \times 5 \times 5 \times 5 = 250$$

(ii) We have, $(5^{-1} \times 2^{-1}) \times 6^{-1}$

$$= \left[\frac{1}{5} \times \frac{1}{2}\right] \times \frac{1}{6} = \left[\frac{1}{10}\right] \times \frac{1}{6} = \frac{1}{60}.$$

$$\left[\because (a)^{-m} = \frac{1}{a^m}\right]$$

5. Find the value of m for which $5^m \div 5^{-3} = 5^5$.

Soln.: We have, $5^m \div 5^{-3} = 5^5$

$$\begin{aligned}&\Rightarrow 5^{m-(-3)} = 5^5 \Rightarrow 5^{m+3} = 5^5 \\&\quad [\because a^m \div a^n = a^{m-n}] \\&\Rightarrow m+3 = 5 \Rightarrow m = 5-3 = 2. \\&\quad [\because a^m = a^n \Rightarrow m = n]\end{aligned}$$

6. Evaluate :

(i) $\left\{\left(\frac{1}{3}\right)^{-1} - \left(\frac{1}{4}\right)^{-1}\right\}^{-1}$ (ii) $\left(\frac{5}{8}\right)^{-7} \times \left(\frac{8}{5}\right)^{-4}$

Soln.: (i) We have,

$$\begin{aligned}\left\{\left(\frac{1}{3}\right)^{-1} - \left(\frac{1}{4}\right)^{-1}\right\}^{-1} &= \left\{\left(\frac{3}{1}\right) - \left(\frac{4}{1}\right)\right\}^{-1} \\&\quad \left[\because \left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m\right] \\&= (3-4)^{-1} = (-1)^{-1} = -1\end{aligned}$$

(ii) We have, $\left(\frac{5}{8}\right)^{-7} \times \left(\frac{8}{5}\right)^{-4}$

$$= \left(\frac{8}{5}\right)^7 \times \left(\frac{5}{8}\right)^4 \quad \left[\because \left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m\right]$$

$$= \frac{(8)^7}{(5)^7} \times \frac{(5)^4}{(8)^4} \quad \left[\because \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}\right]$$

$$= (8)^{7-4} \times (5)^{4-7} \quad \left[\because \frac{a^m}{a^n} = a^{m-n}\right]$$

$$= (8)^3 \times (5)^{-3} = \frac{(8)^3}{(5)^3} = \frac{8 \times 8 \times 8}{5 \times 5 \times 5} = \frac{512}{125}$$

7. Simplify :

(i) $\frac{25 \times t^{-4}}{5^{-3} \times 10 \times t^{-8}} \quad (t \neq 0)$

(ii) $\frac{3^{-5} \times 10^{-5} \times 125}{5^{-7} \times 6^{-5}}$

Soln.: (i) We have, $\frac{25 \times t^{-4}}{5^{-3} \times 10 \times t^{-8}}$

$$= \frac{(5)^2 \times t^{-4}}{(5)^{-3} \times (5)^1 \times (2)^1 \times t^{-8}}$$

$$\begin{aligned}
 &= \frac{(5)^2 \times t^{-4}}{(5)^{-3+1} \times (2)^1 \times t^{-8}} \quad [\because a^m \cdot a^n = a^{m+n}] \\
 &= \frac{(5)^2 \times t^{-4}}{(5)^{-2} \times (2)^1 \times t^{-8}} = \frac{(5)^{2+2} \times t^{-4+8}}{2} \\
 &\quad \left[\because \frac{a^m}{a^n} = a^{m-n} \right] \\
 &= \frac{(5)^4 \times t^4}{2} = \frac{625 \times t^4}{2} = \frac{625t^4}{2}
 \end{aligned}$$

(ii) We have, $\frac{3^{-5} \times 10^{-5} \times 125}{5^{-7} \times 6^{-5}}$

$$\begin{aligned}
 &= \frac{(3)^{-5} \times (2 \times 5)^{-5} \times (5)^3}{(5)^{-7} \times (2 \times 3)^{-5}} \\
 &= \frac{(3)^{-5} \times (2)^{-5} \times (5)^{-5} \times (5)^3}{(5)^{-7} \times (2)^{-5} \times (3)^{-5}} \\
 &\quad [\because (a \cdot b)^m = a^m \cdot b^m]
 \end{aligned}$$

$$\begin{aligned}
 &= (3)^{-5+5} \times (2)^{-5+5} \times (5)^{-5+7+3} \\
 &\quad \left[\because \frac{a^m}{a^n} = a^{m-n}, a^m \cdot a^n = a^{m+n} \right]
 \end{aligned}$$

$$\begin{aligned}
 &= (3)^0 \times (2)^0 \times (5)^5 = 1 \times 1 \times 3125 = 3125 \\
 &\quad [\because a^0 = 1]
 \end{aligned}$$

Exercise 12.2

1. Express the following numbers in standard form.

- (i) 0.0000000000085
- (ii) 0.00000000000942
- (iii) 6020000000000000
- (iv) 0.00000000837
- (v) 31860000000

Soln.: (i) $0.0000000000085 = 8.5 \times 10^{-12}$

(ii) $0.00000000000942 = 9.42 \times 10^{-12}$

(iii) $6020000000000000 = 6.02 \times 10^{15}$

(iv) $0.00000000837 = 8.37 \times 10^{-9}$

(v) $31860000000 = 3.186 \times 10^{10}$

2. Express the following numbers in usual form.

- (i) 3.02×10^{-6}
- (ii) 4.5×10^4
- (iii) 3×10^{-8}
- (iv) 1.0001×10^9
- (v) 5.8×10^{12}
- (vi) 3.61492×10^6

Soln.: (i) We have, $3.02 \times 10^{-6} = 0.00000302$

(ii) We have, $4.5 \times 10^4 = 45000$

(iii) We have, $3 \times 10^{-8} = 0.00000003$

(iv) We have, $1.0001 \times 10^9 = 1000100000$

(v) We have, $5.8 \times 10^{12} = 5800000000000$

(vi) We have, $3.61492 \times 10^6 = 3614920$

3. Express the number appearing in the following statements in standard form.

(i) 1 micron is equal to $\frac{1}{1000000} \text{ m}$

(ii) Charge of an electron is $0.000,000,000,000,000,16 \text{ coulomb.}$

(iii) Size of a bacteria is 0.0000005 m.

(iv) Size of a plant cell is 0.00001275 m.

(v) Thickness of a thick paper is 0.07 mm.

Soln.: (i) Standard form of given statement

$$= 1 \times 10^{-6}.$$

(ii) Standard form of given statement
 $= 1.6 \times 10^{-19}.$

(iii) Standard form of given statement
 $= 5 \times 10^{-7}.$

(iv) Standard form of given statement
 $= 1.275 \times 10^{-5}.$

(v) Standard form of given statement
 $= 7 \times 10^{-2}.$

4. In a stack, there are 5 books each of thickness 20 mm and 5 paper sheets each of thickness 0.016 mm. What is the total thickness of the stack?

Soln.: Total number of books = 5

Thickness of each book = 20 mm

\therefore Thickness of 5 books = $(5 \times 20) \text{ mm} = 100 \text{ mm}$

Thickness of each paper sheet = 0.016 mm

\therefore Thickness of 5 paper sheets = $(5 \times 0.016) \text{ mm} = 0.08 \text{ mm}$

Hence, total thickness of stack

$$= 100 \text{ mm} + 0.08 \text{ mm}$$

$$= 100.08 \text{ mm} = 1.0008 \times 10^2 \text{ mm}$$

EXERCISE

Multiple Choice Questions

Level-1

- | Level-1 | | | |
|---------|---|--|---|
| 1. | The value of $\frac{2^{2001} + 2^{1999}}{2^{2000} - 2^{1998}}$ is | (a) 2
(b) $10/3$
(c) $2^{1000} + 1$
(d) 10 | 8. The value of $(1024)^{-\frac{4}{5}}$ is
(a) $\frac{1}{64}$
(b) $\frac{1}{128}$
(c) $\frac{1}{256}$
(d) $\frac{1}{512}$ |
| 2. | When simplified, $(x^{-1} + y^{-1})^{-1}$ is equal to | (a) $x + y$
(b) $\frac{xy}{x+y}$
(c) xy
(d) $\frac{1}{xy}$ | 9. $(512)^{\frac{-2}{3}} \times \left(\frac{1}{4}\right)^{-3}$ is equal to
(a) 4
(b) $\frac{1}{4}$
(c) 1
(d) 16 |
| 3. | The value of $x + x^x$ when $x = 2$ is | (a) 10
(b) 16
(c) 18
(d) 36 | 10. The value of $(3^2)^3 + \left(\frac{2}{3}\right)^0 + 3^5$ is
(a) 930
(b) 973
(c) 932
(d) 950 |
| 4. | The value of $\left(\frac{243}{32}\right)^{-\frac{4}{5}}$ is | (a) $\frac{81}{16}$
(b) $\frac{16}{81}$
(c) $\frac{4}{9}$
(d) $\frac{9}{4}$ | 11. $16^{\frac{5}{2}} \div 16^{\frac{1}{2}} =$
(a) 250
(b) 256
(c) 255
(d) 200 |
| 5. | The solution of $3^{3x-5} = \frac{1}{9^x}$ is | (a) $\frac{5}{2}$
(b) 5
(c) 1
(d) $\frac{7}{3}$ | 12. $(32)^{\frac{-2}{5}} \div (125)^{\frac{-2}{3}} =$
(a) $\frac{4}{25}$
(b) $\frac{25}{4}$
(c) $\frac{2}{5}$
(d) $\frac{5}{2}$ |
| 6. | Multiplicative inverse of 10^{-100} is | (a) 10^{-100}
(b) $\frac{1}{10^{-100}}$
(c) $(10)^{-10}$
(d) $(10^{-50})^3$ | 13. The value of $(512)^{-2/9}$ is
(a) $\frac{1}{2}$
(b) 2
(c) 4
(d) $\frac{1}{4}$ |
| 7. | $\left(\frac{16}{81}\right)^{\frac{-3}{4}}$ is equal to | (a) $\frac{9}{2}$
(b) $\frac{2}{9}$
(c) $\frac{8}{27}$
(d) $\frac{27}{8}$ | 14. Given that $2^h \times 2^3 = 2^9$, find the value of h .
(a) 3
(b) 6
(c) 8
(d) 12 |
| 8. | $4^{-3/2} + 8^{2/3}$ is equal to | (a) $2\frac{1}{4}$
(b) $4\frac{1}{8}$
(c) $4\frac{1}{4}$
(d) $8\frac{1}{4}$ | 15. $4^{-3/2} + 8^{2/3}$ is equal to
(a) $2\frac{1}{4}$
(b) $4\frac{1}{8}$
(c) $4\frac{1}{4}$
(d) $8\frac{1}{4}$ |

Level-2

Fill in the Blanks

1. $2^5 \times 2^3 = \dots$.
 2. The value of (4^{-3}) is \dots .
 3. The scientific notation for 0.000065 is \dots .
 4. $\left(\frac{25}{21}\right)^{3/2}$ can be written as \dots .

5. The usual form of 6.96×10^5 is

6. If $\sqrt{9x} = \sqrt[3]{9^2}$, then $x =$

7. $(2^5 \div 2^8) \times 2^4$ is

8. $x \frac{a+b-c}{(a-c)(b-c)} \cdot x \frac{b+c-a}{(b-a)(c-a)} \cdot x \frac{c+a-b}{(c-b)(a-b)} =$

9. $\frac{2 \cdot 3^{n+1} + 7 \cdot 3^{n-1}}{3^{n+2} - 2\left(\frac{1}{3}\right)^{1-n}} =$

10. For any two rational numbers a and b , $a^5 \times b^5$ is equal to

True or False

1. $5^0 \times 3^0 = 15$.

2. The value of $\left(\frac{3}{5}\right)^{-3}$ is $\frac{125}{27}$.

3. $(\sqrt{75} \times 4\sqrt{5}) \div \sqrt{45} = \frac{18\sqrt{3}}{19}$.

4. If $\sqrt{a} = 0.2$, then $a^{3/2} = 0.008$

5. $\left(\frac{3}{2}\right)^{-4}$ is reciprocal of $\left(\frac{2}{3}\right)^4$.

6. For a non-zero rational number a , $a^7 \div a^{12}$ is equal to a^5 .

7. If $x = 2$ and $y = 3$, then $x^y = y^x$ holds.

8. $\left(\frac{1}{5}\right)^0$ is equal to 0.

9. $\left(-\frac{2}{5}\right)^7 \div \left(-\frac{2}{5}\right)^5$ is equal to $-\frac{4}{25}$.

10. $\left(\frac{2}{3}\right)^{-3}$ is not equal to $-\frac{8}{27}$.

Match the Following

In this section each question has two matching lists. Choices for the correct combination of elements from List-I and List-II are given as options (a), (b), (c) and (d) out of which one is correct.

1. Simplify and match the following :

List-I

(P) $(3^2 + 2^2) \times \left(\frac{1}{2}\right)^3$

(Q) $(3^2 + 2^2) \times \left(\frac{2}{3}\right)^{-3}$

(R) $\left[\left(\frac{1}{3}\right)^{-3} - \left(\frac{1}{2}\right)^{-3}\right] \div \left[\frac{1}{4}\right]^{-3}$

(S) $(2^2 + 3^2 - 4^2) \div \left(\frac{3}{2}\right)^2$

Code :

P Q R S

(a) 1 2 3 4

(b) 4 1 2 3

(c) 3 4 1 2

(d) 4 2 3 1

2. Solve the following word problems.

List-I

(P) By what number should 5^{-1} be multiplied so that the product may be equal to $(-7)^{-1}$?

(Q) By what number should $(-15)^{-1}$ be divided so that the quotient may be equal to $(-5)^{-1}$?

(R) By what number should $(-6)^{-1}$ be multiplied so that product becomes 9^{-1} ?

(S) By what number should $\left(\frac{-2}{3}\right)^{-3}$ be divided so that the quotient may be equal to $\left(\frac{4}{27}\right)^{-2}$?

Code :

P Q R S

(a) 1 3 2 4

(b) 3 2 4 1

(c) 3 2 1 4

(d) 4 1 2 3

3. Match the following provided that a and b are any rational numbers different from zero and x, y are any rational numbers.

List-I	List-II
(P) $a^x \times a^y$	(1) a^{x-y}
(Q) $a^x \div a^y$	(2) a^{xy}
(R) $(a^x)^y$	(3) a^{x+y}
(S) $(ab)^x$	(4) $a^x \times b^x$

Code :

P	Q	R	S
(a) 1	2	4	3
(b) 3	1	2	4
(c) 4	3	2	1
(d) 3	2	1	4

Assertion & Reason Type

Directions : In each of the following questions, a statement of Assertion is given followed by a corresponding statement of Reason just below it. Of the statements, mark the correct answer as

- (a) If both assertion and reason are true and reason is the correct explanation of assertion.
- (b) If both assertion and reason are true but reason is not the correct explanation of assertion.
- (c) If assertion is true but reason is false.
- (d) If assertion is false but reason is true.

1. **Assertion :** $\left(\frac{5}{7}\right)^0 = 1$

Reason : For any non-zero rational number x , $x^0 = 1$

2. **Assertion :** $(2)^{-3} \div (2)^{-3} = (2)^0$

Reason : $x^{-m} \div x^{-n} = x^{-mn}$

3. **Assertion :** $\left\{\left(\frac{1}{2}\right)^6\right\}^6$ is the reciprocal of 2^{36} .

Reason : $\left\{(a^m)^n\right\} = a^{m \times n}$, for any value of a .

4. **Assertion :** $\sqrt[3]{\frac{27}{216}}$ can be written as $\left(\frac{216}{27}\right)^{1/3}$.

Reason : $a^{1/n} = \sqrt[n]{a}$, where both represents n^{th} root of a .

5. **Assertion :** $\left(-\frac{3}{5}\right)^2 \times \left(-\frac{3}{5}\right)^{-7} = \left(-\frac{3}{5}\right)^{2-7} = \left(-\frac{3}{5}\right)^{-5}$

Reason : $x^n \times y^{-n} = (xy)^{-n}$

Comprehension Type

PASSAGE-I : $a^{-m} \times a^{-n} = a^{-m+(-n)} = a^{-(m+n)}$,

$$a^{-m} \div a^{-n} = a^{(-m+n)}, \quad a^{-m} = \frac{1}{a^m}$$

where m and n are any integers.

1. Evaluate : $\frac{9^{-1} \times 5^3}{3^{-3}}$

- (a) 370 (b) 315 (c) 375 (d) 400

2. Simplify : $\frac{25 \times a^9}{5^{-3} \times 10 \times a^{-18}}$

(a) $\frac{625a^{27}}{12}$ (b) $\frac{625a^5}{20}$ (c) $625a$ (d) $\frac{625a^{27}}{2}$

3. Find the value of m for which $(-3)^{m+1} \times (-3)^5 = (-3)^7$.

- (a) 1 (b) -1 (c) 0 (d) 4

PASSAGE-II : $a^m \times a^n \times a^{-p} = \frac{a^m \times a^n}{a^p}$

1. The value of $6^2 \times 6^{-4} \times 6^8$ is

- (a) 6^6 (b) 6^{-6}
 (c) 6^{12} (d) 6^{-10}

2. Evaluate :

$$\left(\frac{1}{2}\right)^{-4} \times \left(\frac{1}{2}\right)^{-8} \times \left(\frac{1}{2}\right)^2 + \left(\frac{1}{4}\right)^2 \times \left(\frac{1}{4}\right)^{-6} \times \left(\frac{1}{4}\right)^2$$

(a) $\left(\frac{1}{2}\right)^{-10} + \left(\frac{1}{4}\right)^{-2}$ (b) $\left(\frac{1}{2}\right)^{12} \times \left(\frac{1}{4}\right)^{-1}$

(c) $\left(\frac{1}{2}\right)^{-14} + \left(\frac{1}{4}\right)^0$ (d) $\left(\frac{1}{2}\right)^{12}$

3. Find the value of x if $4^{2x-3} = 4^2 \times 2^3 \times 4$.

- (a) 0 (b) 4
 (c) $\frac{15}{4}$ (d) $\frac{-9}{8}$

Subjective Problems

Very Short Answer Type

1. Express the following as a rational number.
(a) $\left(\frac{-3}{5}\right)^3$ (b) $\left(\frac{21}{89}\right)^2$
2. Express the following in power notation.
(a) $\frac{-125}{343}$ (b) $\frac{1}{2401}$
3. Find the product of square of $\frac{-1}{2}$ and the cube of $\frac{-2}{3}$.
4. Simplify : $\left\{6^{-1} + \left(\frac{3}{2}\right)^{-1}\right\}^{-1}$
5. Simplify : $(x^{2^{n-1}} + y^{2^{n-1}})(x^{2^{n-1}} - y^{2^{n-1}})$.
6. If $x^{x\sqrt{x}} = (x^{3/2})^x$, then find x .
7. Find the value of $(-4)^2 \div (2)^5$.
8. Using the laws of exponents, simplify each of the following and express in power notation.
(a) $3^7 \times 3^{-2}$ (b) $2^{-7} \div 2^{-3}$
(c) $(5^2)^{-3}$ (d) $2^{-3} \times (-7)^{-3}$
(e) $\frac{3^{-5}}{4^{-5}}$
9. Simplify : $\left(\frac{x^a}{x^b}\right)^{a+b} \div \left(\frac{x^a}{x^{a-b}}\right)^{a^2/b}$.
10. Evaluate : $(3^2 - 2^2) \times \left(\frac{2}{3}\right)^{-2}$.

Short Answer Type

1. Evaluate : $\left(\frac{-2}{7}\right)^{-4} \times \left(\frac{-5}{7}\right)^2$.
2. If $3^{x+y} = 81$ and $81^{x-y} = 3$, then find the values of x and y .
3. Evaluate : $\left(\frac{-1}{4}\right)^{-3} \times \left(\frac{-1}{4}\right)^{-2}$.

4. If $2^{x-2} = 5^{2-x}$, then find the value of x .
5. By what number should $(-24)^{-1}$ be divided so that the quotient may be 3^{-1} ?
6. Simplify : $\frac{(xyz)^4}{(x^{-2}y^3)^{-3}\left(\frac{1}{z^2}\right)^6}$ ($x \neq 0, y \neq 0, z \neq 0$)
7. By what number should $(-4)^{-2}$ be multiplied so that the product may be equal to 10^{-2} ?
8. Find m so that $\left(\frac{2}{9}\right)^3 \times \left(\frac{2}{9}\right)^{-6} = \left(\frac{2}{9}\right)^{2m-1}$.
9. Write the following numbers using scientific notation :
(a) Two crore fifty three lakh
(b) 980000000000
(c) 0.00000000015
10. The size of a red blood cell is 0.000007 m and the size of the plant cell is 0.00001275 m. Compare their size.

Long Answer Type

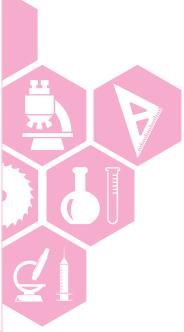
1. Find the largest among $\sqrt[4]{8}, \sqrt{2}, \sqrt[3]{6}$.
2. Express each of the following as power of a rational number with positive exponent :
(a) $5^{-3} \times 5^{-6}$ (b) $\left(\frac{-1}{4}\right)^{-5} \times \left(\frac{-1}{4}\right)^{-7}$
(c) $\left\{\left(\frac{3}{4}\right)^{-1} - \left(\frac{1}{4}\right)^{-1}\right\}^{-1}$
3. Given that $\sqrt[3]{3^x} = 5^{1/4}$ and $\sqrt[4]{5^y} = \sqrt{3}$, then find the value of $2xy$.
4. Simplify :
(a) $(2^{-1} \div 5^{-1})^2 \times \left(\frac{-5}{8}\right)^{-1}$
(b) $(6^{-1} - 8^{-1})^{-1} + (2^{-1} - 3^{-1})^{-1}$
(c) $(5^{-1} \times 3^{-1})^{-1} \div 6^{-1}$
5. If $\frac{10}{3} \times 3^x - 3^{x-1} = 81$. Find the value of x .

Integer Answer Type

In this section, each question, when worked out will result in one integer from 0 to 9 (both inclusive).

1. The value of $(3^0 - 4^0) \times 5^2$ is
 2. The value of x in exponential equation $2^{x-4} = 1$ is equal to
 3. If $\left\{ \left(\frac{1}{3}\right)^{-1} - \left(\frac{1}{4}\right)^{-1} \right\}^m = 1$, then least positive integral value of m is
 4. If $2^{x-1} + 2^{x+1} = 320$, then the value of x is
 5. $(64)^{-2/3} \times (1/4)^{-3}$ equals
 6. $\left(5 \left(8^{1/3} + 27^{1/3} \right) \right)^{1/2} =$
 7. If $\left(\frac{a}{b} \right)^{x-1} = \left(\frac{b}{a} \right)^{x-3}$, then x is equal to
 8. $\left(\frac{x^a}{x^b} \right)^{a+b} \times \left(\frac{x^b}{x^c} \right)^{b+c} \times \left(\frac{x^c}{x^a} \right)^{c+a} =$
 9. $\frac{1}{1+a^{m-n}} + \frac{1}{1+a^{n-m}}$ equals
 10. Given that $4^{n+1} = 256$, find the value of n .
-

CHAPTER
12



Exponents and Powers

Multiple Choice Questions

1. (b) : $\frac{2^{2001} + 2^{1999}}{2^{2000} - 2^{1998}}$

$$= \frac{2^{1999}(2^2 + 1)}{2^{1998}(2^2 - 1)} = 2^{1999 - 1998} \times \frac{5}{3}$$

$$= 2^1 \times \frac{5}{3} = \frac{10}{3}$$

2. (b) : $(x^{-1} + y^{-1})^{-1}$

$$= \left(\frac{1}{x} + \frac{1}{y} \right)^{-1} = \left(\frac{y+x}{xy} \right)^{-1} = \frac{xy}{x+y}$$

3. (a) : $x + x \cdot x^x = x(1 + x^x)$

when $x = 2$, $x(1 + x^x) = 2(1 + 2^2) = 2(1 + 4) = 2 \times 5 = 10$.

4. (b) : $\left(\frac{243}{32} \right)^{-\frac{4}{5}} = \left(\frac{3^5}{2^5} \right)^{-\frac{4}{5}} = \left[\left(\frac{3}{2} \right)^5 \right]^{-\frac{4}{5}}$

$$= \left[\frac{3}{2} \right]^{5 \times -\frac{4}{5}} = \left[\frac{3}{2} \right]^{-4} = \left[\frac{2}{3} \right]^4 = \frac{16}{81}$$

5. (c) : $3^{3x-5} = \frac{1}{9^x}$

$$\Rightarrow 3^{3x-5} = (3^{-2})^x = 3^{-2x}$$

$$\Rightarrow 3x - 5 = -2x$$

$$\Rightarrow 3x + 2x = 5 \Rightarrow 5x = 5 \Rightarrow x = 1$$

6. (b) : Since a^{-m} is multiplicative inverse of a^m .
 \therefore Multiplicative inverse of $10^{-100} = 10^{(-100)}$

$$= 10^{100} = \frac{1}{10^{-100}}$$

7. (d) : $\left(\frac{16}{81} \right)^{-\frac{3}{4}} = \left[\frac{2^4}{3^4} \right]^{-\frac{3}{4}} = \left[\frac{2}{3} \right]^{4 \times -\frac{3}{4}} = \left[\frac{2}{3} \right]^{-3}$

$$= \left[\frac{3}{2} \right]^3 = \frac{27}{8}$$

8. (c) : $(1024)^{-\frac{4}{5}} = (2^{10})^{-\frac{4}{5}}$

$$= 2^{10 \times -\frac{4}{5}} = 2^{-8} = \frac{1}{2^8} = \frac{1}{256}$$

9. (c) : $(512)^{\frac{-2}{3}} \times \left(\frac{1}{4} \right)^{-3}$

$$= (8^3)^{\frac{-2}{3}} \times \left(\frac{1}{4} \right)^{-3} = 8^{3 \times \frac{-2}{3}} \times \left(\frac{1}{4} \right)^{-3}$$

$$= 8^{-2} \times (4)^3 = (2^3)^{-2} \times (2^2)^3$$

$$= 2^{-6} \times 2^6 = 2^{-6+6} = 2^0 = 1$$

10. (b) : $(3^2)^3 + \left(\frac{2}{3} \right)^0 + 3^5$

$$= 3^6 + 1 + 3^5 = 729 + 1 + 243 = 729 + 243 = 973$$

11. (b) : $16^{\frac{5}{2}} \div 16^{\frac{1}{2}} = (16)^{\frac{5}{2} - \frac{1}{2}} = 16^{\frac{4}{2}} = (4^2)^{4/2} = 256$

12. (b) : $(32)^{\frac{-2}{5}} \div (125)^{\frac{-2}{3}} = (2^5)^{\frac{-2}{5}} \div (5^3)^{\frac{-2}{3}}$

$$= 2^{5 \times -\frac{2}{5}} \div 5^{3 \times -\frac{2}{3}}$$

$$= 2^{-2} \div 5^{-2} = \frac{2^{-2}}{5^{-2}} = \frac{5^2}{2^2} = \frac{25}{4}$$

13. (d) : $(512)^{\frac{-2}{9}} = (2^9)^{\frac{-2}{9}} = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

14. (b) : $2^h \times 2^3 = 2^9 \Rightarrow 2^{h+3} = 2^9$

$$\Rightarrow h+3=9$$

$$\therefore h=9-3=6 \Rightarrow h=6$$

15. (b) : $4^{-3/2} + 8^{2/3} = [2^2]^{-3/2} + [2^3]^{2/3}$

$$= 2^{-3} + 2^2 = \frac{1}{2^3} + 4 = \frac{1}{8} + 4 = 4\frac{1}{8}$$

16. (c) : $\frac{10}{3} \times 3^x - 3^{x-1} = 81$

$$\Rightarrow 10 \times 3^{x-1} - 3^{x-1} = 81$$

$$\Rightarrow 3^{x-1}[10-1] = 81$$

$$\Rightarrow 3^{x-1} \times 9 = 81$$

$$\Rightarrow 3^{x-1} = \frac{81}{9} = 9 = 3^2$$

$$\Rightarrow 3^{x-1} = 3^2 \Rightarrow x-1=2$$

$$\Rightarrow x=3.$$

17. (a) : (I) $1^4 = 1$, (II) $4^0 = 1$, (III) $0^4 = 0$, (IV) $4^1 = 4$
 \therefore (I) and (II) are equal.

18. (a) : $15240000 = 1.524 \times 10^7$

(Decimal is shifted to left by 7 places).

19. (d) : $\frac{4r}{(r^2 m)^2} = \frac{4r}{r^4 m^2} = \frac{4}{r^3 m^2}$

20. (c) : $\left(\frac{16}{81}\right)^{\frac{3}{4}} = \left(\frac{2^4}{3^4}\right)^{\frac{3}{4}} = \left(\frac{2}{3}\right)^{4 \times \frac{3}{4}} = \left(\frac{2}{3}\right)^3 = \frac{8}{27}$

21. (a) : $(8^0 - 3^0) \times (8^0 + 3^0)$

$$= (1-1)(1+1) = 0 \times 2 = 0$$

22. (b) : $8^{\frac{4}{3}} \times 2^{-1} = [2^3]^{\frac{4}{3}} \times 2^{-1} = 2^4 \times 2^{-1}$
 $= 2^{4-1} = 2^3 = 8.$

23. (d) : (a) $\left(y^{\frac{2}{3}}\right)^9 = y^{\frac{2}{3} \times 9} = y^6$

(b) $\left(\sqrt{y^6}\right)^2 = y^{\frac{6}{2} \times 2} = y^6$

(c) $\sqrt[3]{y^{18}} = y^{\frac{18}{3}} = y^6$

(d) $y^{\frac{12}{3}} = y^4$

\therefore (d) is not equal to y^6 .

24. (b) : $\left(\frac{1}{3}\right)^3 \times \left(\frac{-2}{5}\right)^2 \times \left(\frac{-3}{2}\right)^3$

$$= \frac{1}{3^3} \times \frac{(-2)^2}{5^2} \times \frac{(-3)^3}{(2)^3}$$

$$= (-1) \times 2^{2-3} \times 3^{3-3} \times 5^{-2}$$

$$= -1 \times 2^{-1} \times 3^0 \times 5^{-2} = \frac{-1}{2 \times 25} = \frac{-1}{50}$$

25. (b) : $9x^6y^2 \div 3x^3y = \frac{9x^6y^2}{3x^3y} = \frac{9}{3} x^{6-3} \cdot y^{2-1}$
 $= 3x^3y$

26. (c) : $3\sqrt[3]{2} \times 7\sqrt[3]{6} \times 5\sqrt[3]{18}$

$$= 3 \times (2)^{\frac{1}{3}} \times 7 \times (6)^{\frac{1}{3}} \times 5 \times (18)^{\frac{1}{3}}$$

$$= 105 \times (2 \times 6 \times 18)^{\frac{1}{3}} = 105 \times (216)^{\frac{1}{3}}$$

$$= 105 \times (6^3)^{\frac{1}{3}} = 105 \times (6)^{3 \times \frac{1}{3}} = 105 \times 6 = 630$$

27. (b) : $x^y = y^x$

$$\Rightarrow \frac{x^y}{y^x} = 1 \text{ or } y = x^{\frac{y}{x}} \quad \dots\dots \text{(i)}$$

Now, $\left(\frac{x}{y}\right)^{\frac{x}{y}} = \left(\frac{x}{x^{y/x}}\right)^{\frac{x}{y}} \quad \text{(using (i))}$

$$= \frac{x^{x/y}}{x^{y/x} \times x/y} = \frac{x^{x/y}}{x} = x^{x/y-1}$$

28. (b) : $\frac{(5)^{0.25} \times (125)^{0.25}}{(256)^{0.10} \times (256)^{0.15}} = \frac{(5)^{\frac{25}{100}} \times (125)^{\frac{25}{100}}}{(256)^{\frac{10}{100}} \times (256)^{\frac{15}{100}}}$

$$= \frac{(5)^{\frac{1}{4}} \times (5^3)^{\frac{1}{4}}}{(256)^{\frac{25}{100}}} = \frac{\frac{1}{5^4} \times \frac{3}{5^4}}{(4^4)^{\frac{1}{4}}} = \frac{\frac{1+3}{4}}{4^{\frac{4}{4}}} = \frac{5}{4}$$

29. (c) :
$$\frac{\sqrt[3]{81} + \sqrt[3]{-192} + \sqrt[3]{375}}{\sqrt[3]{24}}$$

$$= \frac{(3^3 \cdot 3)^{\frac{1}{3}} + (-1)^{\frac{1}{3}} \times (4^3)^{\frac{1}{3}} \times 3^{\frac{1}{3}} + (5^3)^{\frac{1}{3}} \times 3^{\frac{1}{3}}}{[2^3 \cdot (3)]^{\frac{1}{3}}}$$

$$= \frac{4 \cdot 3^{\frac{1}{3}}}{2 \cdot 3^{\frac{1}{3}}} = 2$$

30. (b) :
$$\left\{(8)^{-2/3}\right\}^{\frac{(27)^{\frac{1}{3}}}{2}} = \left\{(2^3)^{-2/3}\right\}^{\frac{(3^3)^{\frac{1}{3}}}{2}}$$

$$= \left[(2^3)^{-2/3}\right]^{3/2} = 2^{3 \times \frac{-2}{3} \times \frac{3}{2}} = 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

Fill in the Blanks

1. $2^8 : 2^5 \times 2^3 = 2^{5+3} = 2^8$

2. $\frac{1}{64} : 4^{-3} = \frac{1}{4^3} = \frac{1}{64}.$

3. $6.5 \times 10^{-5} : 0.000065 = \frac{65}{1000000} = 6.5 \times 10^{-5}.$

4. $\sqrt{\left(\frac{25}{21}\right)^3} : \left(\frac{25}{21}\right)^{3/2} = \sqrt{\left(\frac{25}{21}\right)^3}$

5. $696000 : 6.96 \times 10^5 = 6.96 \times 100000 = 696000$

6. $3^{2/3} : \sqrt{9x} = \sqrt[3]{9^2} \Rightarrow (9x)^{\frac{1}{2}} = 9^{2/3}$

$$\Rightarrow 9^{\frac{1}{2}} \cdot x^{\frac{1}{2}} = 9^{2/3} \Rightarrow x^{\frac{1}{2}} = 9^{\frac{2}{3} - \frac{1}{2}}$$

$$\Rightarrow x^{\frac{1}{2}} = 9^{\frac{4-3}{6}} \Rightarrow x^{\frac{1}{2}} = 9^{\frac{1}{6}}$$

$$\Rightarrow x = 9^{2/6} = 9^{\frac{1}{3}} = (3^2)^{\frac{1}{3}} \Rightarrow x = 3^{2/3}$$

7. $2 : \left(\frac{2^5}{2^8}\right) \times 2^4 = 2^{-3} \times 2^4 = 2$

8. 1: $x^{\frac{a+b-c}{(a-c)(b-c)}} \cdot x^{\frac{b+c-a}{(b-a)(c-a)}} \cdot x^{\frac{c+a-b}{(c-b)(a-b)}}$

$$= x^{\left(\frac{-(a-b)(a+b-c)-(b-c)(b+c-a)-(c-a)(c+a-b)}{(a-b)(b-c)(c-a)}\right)}$$

$$= x^0 = 1$$

9. 1 :
$$\frac{2 \cdot 3^n \cdot 3 + 7 \cdot 3^n \cdot 3^{-1}}{3^n \cdot 3^2 - 2 \cdot 3^{-1} \cdot 3^n}$$

$$= \frac{3^n \left[6 + \frac{7}{3} \right]}{3^n \left[9 - \frac{2}{3} \right]} = \frac{\frac{18+7}{3}}{\frac{27-2}{3}} = \frac{25}{3} \times \frac{3}{25} = 1$$

10. $(ab)^5 : a^5 \times b^5 = (ab)^5.$

True or False

1. **False** : $5^0 \times 3^0 = 1 \times 1 = 1 \neq 15.$

2. **True** : $\left(\frac{3}{5}\right)^{-3} = \left(\frac{5}{3}\right)^3 = \frac{125}{27}.$

3. **False** : $\sqrt{75} \times 4\sqrt{5} \div \sqrt{45} = \frac{5\sqrt{3} \times 4\sqrt{5}}{3\sqrt{5}} \neq \frac{18\sqrt{3}}{19}$

4. **True** : $a^{\frac{1}{2}} = 0.2$

Taking cube on both sides,

$$\therefore a^{\frac{3}{2}} = (0.2)^3 = 0.008$$

5. **False** : Reciprocal of $\left(\frac{2}{3}\right)^4$ is either $\left(\frac{2}{3}\right)^{-4}$ or $\left(\frac{3}{2}\right)^4$.

6. **False** : $a^7 \div a^{12} = a^{7-12} = a^{-5}.$

7. **False** : $x^y = 2^3 = 8, y^x = 3^2 = 9$

So, $x^y \neq y^x$.

8. **False** : $\left(\frac{1}{5}\right)^0 = 1$ as $a^0 = 1$.

9. **False** : $\left(\frac{-2}{5}\right)^7 \div \left(\frac{-2}{5}\right)^5 = \left(\frac{-2}{5}\right)^{7-5}$
 $= \left(\frac{-2}{5}\right)^2 = \frac{4}{25}.$

10. **True** : $\left(\frac{2}{3}\right)^{-3} = \left(\frac{3}{2}\right)^3 = \frac{27}{8}$

$$\therefore \left(\frac{2}{3}\right)^{-3} = \frac{27}{8}.$$

Match the Following

1. (c) : P → 3; Q → 4; R → 1; S → 2

$$(P) (3^2 + 2^2) \times \left(\frac{1}{2}\right)^3 = (9 + 4) \times \frac{1}{8} = \frac{13}{8}$$

$$(Q) (3^2 + 2^2) \times \left(\frac{2}{3}\right)^{-3} = (9 + 4) \times \frac{3^3}{2^3} = \frac{13 \times 27}{8} = \frac{351}{8}$$

$$(R) \left[\left(\frac{1}{3}\right)^{-3} - \left(\frac{1}{2}\right)^{-3} \right] \div \left[\frac{1}{4}\right]^{-3} = [3^3 - 2^3] \div [4]^3 = [27 - 8] \div 64 = \frac{19}{64}$$

$$(S) (2^2 + 3^2 - 4^2) \div \left(\frac{3}{2}\right)^2 = [4 + 9 - 16] \div \left(\frac{3}{2}\right)^2 = -3 \div \frac{9}{4} = -3 \times \frac{4}{9} = -\frac{4}{3}$$

2. (d) : P → 4; Q → 1; R → 2; S → 3

$$(P) 5^{-1} \times x = (-7)^{-1}$$

$$\Rightarrow x = \left(\frac{-7}{5}\right)^{-1} = \left(\frac{5}{-7}\right) = \frac{-5}{7}$$

$$(Q) (-15)^{-1} \div x = (-5)^{-1}$$

$$\Rightarrow \frac{(-15)^{-1}}{x} = (-5)^{-1}$$

$$\Rightarrow \frac{(-15)^{-1}}{(-5)^{-1}} = x \Rightarrow x = \frac{-5}{-15}$$

$$\Rightarrow x = \frac{1}{3}$$

$$(R) (-6)^{-1} \times x = 9^{-1}$$

$$\Rightarrow x = \left(\frac{9}{-6}\right)^{-1} = \left(\frac{3}{-2}\right)^{-1} = \frac{-2}{3}$$

$$(S) \left(\frac{-2}{3}\right)^{-3} \div x = \left(\frac{4}{27}\right)^{-2} \Rightarrow \left(\frac{-2}{3}\right)^{-3} \div \frac{2^{-4}}{3^{-6}} = x$$

$$\Rightarrow x = \frac{(-2)^{-3}}{3^{-3}} \times \frac{3^{-6}}{2^{-4}} = (-1)^{-3} \frac{(2)^{-3+4}}{3^{-3+6}} = \frac{-2}{27}$$

3. (b) : P → 3; Q → 1; R → 2; S → 4

$$(P) a^x \times a^y = a^{x+y}$$

$$(Q) a^x \div a^y = a^{x-y}$$

(R) $(a^x)^y = a^{xy}$

(S) $(ab)^x = a^x \times b^x$

Assertion & Reason Type

1. (a) : As $\left(\frac{5}{7}\right)^0 = 1$, since $x^0 = 1$ for all non zero x .

Assertion : True ; **Reason** : True and is the correct explanation of assertion.

2. (c) : $(2)^{-3} \div 2^{-3} = 2^{-3 - (-3)} = 2^{-3+3} = 2^0$.
 $x^{-m} \div x^{-n} = x^{-m - (-n)} = x^{-m+n} = x^{n-m}$

Assertion : True ; **Reason** : False.

3. (b) : Reciprocal of $2^{36} = 2^{-36}$ or $\left(\frac{1}{2}\right)^{36}$.

$$= \left(\left(\frac{1}{2}\right)^6\right)^6 \text{ as } (a^m)^n = a^{m \times n} \text{ for any value of } a$$

∴ **Assertion** : True ; **Reason** : True but is not the correct explanation of assertion.

4. (d) : $\sqrt[3]{\frac{27}{216}} = \left(\frac{27}{216}\right)^{\frac{1}{3}}$

Assertion : False; **Reason** : True.

5. (c) : $\left(\frac{-3}{5}\right)^2 \times \left(\frac{-3}{5}\right)^{-7} = \left(\frac{-3}{5}\right)^{2-7} = \left(\frac{-3}{5}\right)^{-5}$

$$\text{But } x^n \times y^{-n} = \frac{x^n}{y^n} = \left(\frac{x}{y}\right)^n$$

Assertion : True ; **Reason** : False.

Comprehension Type

PASSAGE-I

1. (c) : $\frac{9^{-1} \times 5^3}{3^{-3}} = \frac{1}{9} \times 27 \times 125 = 375$

2. (d) : We have, $\frac{25 \times a^9}{5^{-3} \times 10 \times a^{-18}}$
 $= \frac{5^2 \times a^9}{5^{-3} \times 2 \times 5 \times a^{-18}}$
 $= \frac{5^2 \times a^9}{5^{-3+1} \times 2 \times a^{-18}} = \frac{5^{2+2} \times a^{9+18}}{2}$
 $= \frac{5^4 \times a^{27}}{2} = \frac{625a^{27}}{2}$

3. (a) : $(-3)^{m+1} \times (-3)^5 = (-3)^7$

$$\Rightarrow (-3)^{m+1+5} = (-3)^7$$

$$\Rightarrow m+6=7 \Rightarrow m=1$$

PASSAGE-II

1. (a) : $6^2 \times 6^{-4} \times 6^8 = \frac{6^2 \times 6^8}{6^4} = \frac{6^{10}}{6^4} = 6^{10-4} = 6^6$

2. (a) : $\left(\frac{1}{2}\right)^{-4} \times \left(\frac{1}{2}\right)^{-8} \times \left(\frac{1}{2}\right)^2 + \left(\frac{1}{4}\right)^2 \times \left(\frac{1}{4}\right)^{-6} \times \left(\frac{1}{4}\right)^2$

$$= \frac{\left(\frac{1}{2}\right)^2}{\left(\frac{1}{2}\right)^{12}} + \frac{\left(\frac{1}{4}\right)^4}{\left(\frac{1}{4}\right)^6} = \left(\frac{1}{2}\right)^{-10} + \left(\frac{1}{4}\right)^{-2}$$

3. (c) : $4^{2x-3} = 4^2 \times 2^3 \times 4$

$$\Rightarrow 4^{2x-3} = 4^3 \times 2^3$$

$$\Rightarrow (2^2)^{2x-3} = (2^2)^3 \times 2^3$$

$$\Rightarrow 2^{4x-6} = 2^9$$

$$\Rightarrow 4x - 6 = 9 \Rightarrow 4x = 15 \Rightarrow x = \frac{15}{4}$$

Subjective Problems

Very Short Answer Type

1. (a) $\left(\frac{-3}{5}\right)^3 = \frac{(-3) \times (-3) \times (-3)}{5 \times 5 \times 5} = \frac{-27}{125}$

(b) $\left(\frac{21}{89}\right)^2 = \frac{21 \times 21}{89 \times 89} = \frac{441}{7921}$

2. (a) The given number is $\frac{-125}{343}$

$$\frac{-125}{343} = \frac{(-5) \times (-5) \times (-5)}{7 \times 7 \times 7} = \frac{(-5)^3}{(7)^3} = \left(\frac{-5}{7}\right)^3$$

(b) $\frac{1}{2401} = \frac{1}{7 \times 7 \times 7 \times 7} = \frac{1}{7^4} = \left(\frac{1}{7}\right)^4$

3. Square of $\frac{-1}{2} = \left(\frac{-1}{2}\right)^2 = \frac{1}{4}$

and cube of $\frac{-2}{3} = \left(\frac{-2}{3}\right)^3 = \frac{-8}{27}$

Therefore, the required product is

$$\frac{1}{4} \times \left(\frac{-8}{27}\right) = \frac{-8}{4 \times 27} = \frac{-2}{27}$$

4. We have, $\left\{6^{-1} + \left(\frac{3}{2}\right)^{-1}\right\}^{-1} = \left\{6^{-1} + \frac{2}{3}\right\}^{-1}$

$$= \left\{\frac{1}{6} + \frac{2}{3}\right\}^{-1} = \left(\frac{1+4}{6}\right)^{-1} = \left(\frac{5}{6}\right)^{-1} = \frac{6}{5}$$

5. In the given expression, we assume that $x^{2^{n-1}} = a$ and $y^{2^{n-1}} = b$

$$\text{Then, it becomes } (a+b)(a-b) = a^2 - b^2$$

$$= [x^{2^{n-1}}]^2 - [y^{2^{n-1}}]^2 = x^{2^{n-1} \times 2^1} - y^{2^{n-1} \times 2^1}$$

$$= x^{2^{n-1+1}} - y^{2^{n-1+1}} = x^{2^n} - y^{2^n}$$

6. $x^{x\sqrt{x}} = \left(x^{3/2}\right)^x \Rightarrow x^{x\sqrt{x}} = x^{\frac{3}{2}x}$

$$\Rightarrow x\sqrt{x} = \frac{3}{2}x \Rightarrow \sqrt{x} = \frac{3}{2} \Rightarrow x = \frac{9}{4}$$

7. $(-4)^2 \div (2)^5 = \frac{(-2)^4}{2^5} = 2^{4-5} = 2^{-1} = \frac{1}{2}$

8. Using the law of exponents, we have

(a) $3^7 \times 3^{-2} = 3^{7+(-2)} = 3^5 \quad [\because a^m \times a^n = a^{m+n}]$

(b) $2^{-7} \div 2^{-3} = \frac{2^{-7}}{2^{-3}} = 2^{-7-(-3)} \quad \left[\because \frac{a^m}{a^n} = a^{m-n}\right]$

$$= 2^{-7+3} = 2^{-4}$$

(c) $(5^2)^{-3} = 5^{2 \times (-3)} = 5^{-6} \quad [\because (a^m)^n = a^{mn}]$

(d) $2^{-3} \times (-7)^{-3} = (2 \times (-7))^{-3} \quad [\because a^n \times b^n = (ab)^n]$
 $= (-14)^{-3}$

(e) $\frac{3^{-5}}{4^{-5}} = \left(\frac{3}{4}\right)^{-5} \quad \left[\because \frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n\right]$

9. $\left(\frac{x^a}{x^b}\right)^{a+b} \div \left(\frac{x^a}{x^{a-b}}\right)^{\frac{a^2}{b}} = x^{(a-b)(a+b)} \div (x^{a-a+b})^{\frac{a^2}{b}}$

$$= x^{a^2-b^2} \div (x^b)^{\frac{a^2}{b}} = x^{a^2-b^2} \div x^{a^2}$$

$$= x^{a^2-b^2-a^2} = x^{-b^2}$$

10. $(3^2 - 2^2) \times \left(\frac{2}{3}\right)^{-2}$
 $= (9 - 4) \times \left(\frac{3}{2}\right)^2 = (5) \times \frac{9}{4} = \frac{45}{4}$

Short Answer Type

1. $\left(\frac{-2}{7}\right)^{-4} \times \left(\frac{-5}{7}\right)^2 = \left(\frac{7}{-2}\right)^4 \times \left(\frac{-5}{7}\right)^2$
 $= \left(\frac{-7}{2}\right)^4 \times \left(\frac{-5}{7}\right)^2 = \frac{7^4 \times 5^2}{2^4 \times 7^2} = \frac{49 \times 25}{16} = \frac{1225}{16}$

2. We have, $3^{x+y} = 81 = 3^4$ and $81^{x-y} = (3^4)^{x-y} = 3^1$
 $\Rightarrow x + y = 4$ (i)
and $4x - 4y = 1$ (ii)
Applying $4 \times$ (i) + (ii), we get
 $\Rightarrow 4x + 4y + 4x - 4y = 16 + 1$
 $\Rightarrow 8x = 17 \Rightarrow x = \frac{17}{8}$

Substituting the value of x in (i), we get,
 $y = \frac{15}{8}$

3. $\left(\frac{-1}{4}\right)^{-3} \times \left(\frac{-1}{4}\right)^{-2} = \left(\frac{4}{-1}\right)^3 \times \left(\frac{4}{-1}\right)^2 = (-4)^3 \times (-4)^2$
 $= (-4)^{3+2} = (-4)^5 = -4^5 = -1024$

4. $2^{x-2} = 5^{2-x} \Rightarrow 2^{x-2} = \frac{1}{5^{x-2}} \Rightarrow 10^{x-2} = 1$
 $\Rightarrow \frac{10^x}{10^2} = 1 \Rightarrow 10^x = 10^2 \Rightarrow x = 2.$

5. Let the required number be x . Then,
 $(-24)^{-1} \div x = 3^{-1}$

$$\Rightarrow \frac{(-24)^{-1}}{x} = 3^{-1} \Rightarrow \frac{-24}{x} = \frac{1}{3} \Rightarrow \frac{1}{-24x} = \frac{1}{3}$$

$$\Rightarrow 3 = -24x \Rightarrow x = \frac{3}{-24} \Rightarrow x = -\frac{1}{8}$$

6. We have, $\frac{(xyz)^4}{(x^{-2}y^3)^{-3}\left(\frac{1}{z^2}\right)^6} = \frac{x^4y^4z^4}{x^6y^{-9}z^{-12}}$
 $= x^{4-6}y^{4-(-9)}z^{4-(-12)} = x^{-2}y^{13}z^{16} = \frac{y^{13}z^{16}}{x^2}$

7. Let $(-4)^{-2}$ be multiplied by x to get 10^{-2} . Then,
 $x \times (-4)^{-2} = 10^{-2}$
 $\Rightarrow x = 10^{-2} \div (-4)^{-2}$
 $\Rightarrow x = 10^{-2} \times \frac{1}{(-4)^{-2}} \Rightarrow x = \frac{10^{-2}}{(-4)^{-2}} = \left(\frac{10}{-4}\right)^{-2}$

$$\Rightarrow x = \left(\frac{-4}{10}\right)^2 = \frac{16}{100} = \frac{4}{25}$$

8. We have, $\left(\frac{2}{9}\right)^3 \times \left(\frac{2}{9}\right)^{-6} = \left(\frac{2}{9}\right)^{2m-1}$
 $\Rightarrow \left(\frac{2}{9}\right)^{3-6} = \left(\frac{2}{9}\right)^{2m-1}$
 $\Rightarrow \left(\frac{2}{9}\right)^{-3} = \left(\frac{2}{9}\right)^{2m-1}$
 $\Rightarrow 2m - 1 = -3 \Rightarrow 2m = -3 + 1$
 $\Rightarrow 2m = -2 \Rightarrow m = -1$

9. (a) Two crore fifty three lakh = 2,53,00,000
 $= 253 \times 10^5 = 2.53 \times 10^7$
(b) 98000000000 = $98 \times 10^9 = 9.8 \times 10^{10}$
(c) $0.00000000015 = \frac{15}{100000000000}$
 $= \frac{15}{10^{11}} = 1.5 \times 10^{-10}$

10. We have,
Size of red blood cell = 0.000007 m = 7×10^{-6} m
Size of plant cell = 0.00001275 m = 1.275×10^{-5} m
 $\therefore \frac{\text{Size of red blood cell}}{\text{Size of plant cell}} = \frac{7 \times 10^{-6}}{1.275 \times 10^{-5}} = \frac{7 \times 10^{-6+5}}{1.275} = \frac{7 \times 10^{-1}}{1.275} = \frac{0.7}{1.275} = \frac{0.7}{1.3} \approx \frac{1}{2}$
So, a red blood cell is approximately half of a plant cell in size.

Long Answer Type

1. We will either make base same or exponent same to compare the numbers.
Given exponents are of order 4, 2, 3 respectively.
Their L.C.M. is 12.

Changing each to a exponent of order 12,
we get

$$\sqrt[4]{8} = (8)^{\frac{1}{4}} = (8)^{\frac{1}{4} \times \frac{3}{3}} = \sqrt[12]{8^3} = \sqrt[12]{512}$$

$$\sqrt{2} = (2)^{\frac{1}{2}} = 2^{\frac{1}{2} \times \frac{6}{6}} = \sqrt[12]{2^6} = \sqrt[12]{64}$$

$$\sqrt[3]{6} = (6)^{\frac{1}{3}} = (6)^{\frac{1}{3} \times \frac{4}{4}} = \sqrt[12]{6^4} = \sqrt[12]{1296}$$

$$\therefore \sqrt[12]{1296} > \sqrt[12]{512} > \sqrt[12]{64} \text{ or } \sqrt[3]{6} > \sqrt[4]{8} > \sqrt{2}$$

2. (a) We have,

$$5^{-3} \times 5^{-6} = \frac{1}{5^3} \times \frac{1}{5^6} = \frac{1}{5^3 \times 5^6} = \frac{1}{5^{3+6}} = \frac{1}{5^9}$$

$$(b) \text{ We have, } \left(\frac{-1}{4}\right)^{-5} \times \left(\frac{-1}{4}\right)^{-7}$$

$$\begin{aligned} &= \frac{1}{\left(\frac{-1}{4}\right)^5} \times \frac{1}{\left(\frac{-1}{4}\right)^7} = \frac{1}{(-1)^5} \times \frac{1}{(-1)^7} = \frac{1}{4^5} \times \frac{1}{4^7} \\ &= \frac{4^5 \times 4^7}{(-1) \times (-1)} = \frac{4^{5+7}}{1} = 4^{12} \end{aligned}$$

$$(c) \text{ We have, } \left\{ \left(\frac{3}{4}\right)^{-1} - \left(\frac{1}{4}\right)^{-1} \right\}^{-1} = \left(\frac{1}{\frac{3}{4}} - \frac{1}{\frac{1}{4}} \right)^{-1}$$

$$= \left(\frac{4}{3} - \frac{4}{1} \right)^{-1} = \left(\frac{4 - 3 \times 4}{3} \right)^{-1} = \left(\frac{-8}{3} \right)^{-1} = -\frac{3}{8}$$

$$3. \sqrt[3]{3^x} = 5^{1/4} \Rightarrow 3^{x/3} = 5^{1/4} \quad \dots(i)$$

$$\text{and } \sqrt[4]{5^y} = \sqrt{3} \Rightarrow 5^{y/4} = 3^{1/2}$$

$$\Rightarrow 5 = 3^{\frac{1}{2} \times \frac{4}{y}} \quad \dots(ii)$$

Putting the value of 5 from equation (ii) in equation (i).

$$3^{x/3} = 3^{\left(\frac{1}{2} \times \frac{4}{y}\right) \frac{1}{4}} \Rightarrow \frac{x}{3} = \frac{1}{2} \times \frac{4}{y} \times \frac{1}{4}$$

$$\Rightarrow 2xy = 3$$

4. (a) We have,

$$(2^{-1} \div 5^{-1})^2 \times \left(\frac{-5}{8}\right)^{-1} = \left(\frac{1}{2} \div \frac{1}{5}\right)^2 \times \frac{1}{\left(\frac{-5}{8}\right)}$$

$$= \left(\frac{1}{2} \times 5\right)^2 \times \left(\frac{8}{-5}\right) = \left(\frac{5}{2}\right)^2 \times \left(\frac{8}{-5}\right)$$

$$= \frac{5^2}{2^2} \times \frac{8}{-5} = \frac{5}{4} \times \frac{8}{-1} = \frac{5}{1} \times \frac{2}{-1} = -10$$

- (b) We have, $(6^{-1} - 8^{-1})^{-1} + (2^{-1} - 3^{-1})^{-1}$

$$= \left(\frac{1}{6} - \frac{1}{8}\right)^{-1} + \left(\frac{1}{2} - \frac{1}{3}\right)^{-1}$$

$$= \left(\frac{4-3}{24}\right)^{-1} + \left(\frac{3-2}{6}\right)^{-1}$$

$$= \left(\frac{1}{24}\right)^{-1} + \left(\frac{1}{6}\right)^{-1} = \frac{1}{\frac{1}{24}} + \frac{1}{\frac{1}{6}} = \frac{24}{1} + \frac{6}{1} = 30$$

- (c) We have, $(5^{-1} \times 3^{-1})^{-1} \div 6^{-1}$

$$= \left(\frac{1}{5} \times \frac{1}{3}\right)^{-1} \div \frac{1}{6}$$

$$= \left(\frac{1}{15}\right)^{-1} \div \frac{1}{6} = \frac{1}{1} \div \frac{1}{6} = 15 \div \frac{1}{6} = 15 \times \frac{6}{1} = 90$$

5. We have, $\frac{10}{3} \times 3^x - 3^{x-1} = 81$

$$\Rightarrow \frac{10}{3} \times 3^x - \frac{3^x}{3} = 81$$

$$\Rightarrow \frac{10 \times 3^x - 3^x}{3} = 81$$

$$\Rightarrow 3^x(10 - 1) = 81 \times 3$$

$$\Rightarrow 3^x \times 9 = 81 \times 3$$

$$\Rightarrow 3^x = \frac{81 \times 3}{9}$$

$$\Rightarrow 3^x = 9 \times 3 \Rightarrow 3^x = 3^3 \Rightarrow x = 3$$

Integer Answer Type

1. (0) : $(3^0 - 4^0) \times 5^2 = (1 - 1) \times 5^2 = 0.$

2. (4) : $2^{x-4} = 1 \Rightarrow 2^{x-4} = 2^0$

$$\Rightarrow x - 4 = 0 \Rightarrow x = 4$$

3. (2) : $\left[\left(\frac{1}{3} \right)^{-1} - \left(\frac{1}{4} \right)^{-1} \right]^m = 1$

$$\Rightarrow [3 - 4]^m = 1$$

$$\Rightarrow [-1]^m = 1$$

The least value for which it is true is $m = 0,$

But zero is not a positive integer.

$$\therefore [-1]^m = 1^2 \Rightarrow m = 2$$

4. (7) : $2^{x-1} + 2^{x+1} = 320$

$$\Rightarrow 2^x [2^{-1} + 2] = 320$$

$$\Rightarrow 2^x \left[\frac{1}{2} + 2 \right] = 320 \Rightarrow 2^x \left[\frac{1+4}{2} \right] = 320$$

$$\Rightarrow 2^x = \frac{320 \times 2}{5} = 128$$

$$\Rightarrow 2^x = 2^7 \Rightarrow x = 7$$

5. (4) : $(64)^{-2/3} \times \left(\frac{1}{4} \right)^{-3}$

$$= (4^3)^{-2/3} \times \left(\frac{1}{4} \right)^{-3} = 4^{-\frac{2 \times 3}{3}} \times (4)^3$$

$$= 4^{-2} \times 4^3 = 4^{3-2} = 4.$$

6. (5) : $\left(5 \left((2^3)^{\frac{1}{3}} + ((3)^3)^{\frac{1}{3}} \right) \right)^{\frac{1}{2}}$

$$= [5 (2 + 3)]^{\frac{1}{2}} = (5 \times 5)^{\frac{1}{2}} = 5$$

7. (2) : $\left(\frac{a}{b} \right)^{x-1} = \left(\frac{b}{a} \right)^{x-3} = \left(\frac{a}{b} \right)^{-(x-3)}$

$$\Rightarrow x - 1 = -x + 3$$

$$\Rightarrow 2x = 3 + 1 \Rightarrow 2x = 4 \Rightarrow x = 2$$

8. (1) : $x^{(a-b)(a+b)} \times x^{(b-c)(b+c)} \times x^{(c-a)(c+a)}$

$$= x^{a^2-b^2} \times x^{b^2-c^2} \times x^{c^2-a^2}$$

$$= x^{a^2-b^2+b^2-c^2+c^2-a^2} = x^0 = 1$$

9. (1) : $\frac{1}{1 + \frac{a^m}{a^n}} + \frac{1}{1 + \frac{a^n}{a^m}} = \frac{a^n}{a^n + a^m} + \frac{a^m}{a^m + a^n}$

$$= \frac{a^m + a^n}{a^m + a^n} = 1$$

10. (3) : $4^{n+1} = 256$

$$\Rightarrow 4^{n+1} = 4^4 \Rightarrow n + 1 = 4$$

$$\Rightarrow n = 4 - 1 = 3$$

