

CHAPTER 9

Applications of Trigonometry

Introduction
Early astronomers have used trigonometry to calculate the distances from the Earth to the planets and stars. Trigonometry is also used in geography and in navigation. Maps are constructed with the help of trigonometry to determine the position of island in relation to the longitudes and latitudes.
In this chapter, we shall applying the trigonometric results to discuss problems regarding heights and distances of various objects without measuring them.
There are some terms which will be used in this chapter.
Line of Sight
The line of sight is drawn from eye of an observer to the point in the object viewed by the observer. If $O$ is the eye of an observer and $A$ and $B$ are objects, then $O A$ and $O B$ is called the line of sight.


Angle of Elevation
The angle of elevation of an object viewed is the angle formed by the line of sight with the horizontal when the point being viewed is above the horizontal level.
In the above given figure $\angle A O X$ is the angle of elevation.
Angle of Depression
The angle of depression of an object viewed is the angle formed by the line of sight with the horizontal when the point being viewed is below the horizontal level.
In the given above figure $\angle X O B$ is called the angle of depression.

## SOLVED EXAMPLES

1. Find the height of a tower if the angle of elevation of top of tower is $60^{\circ}$ and the horizontal distance from eye to the foot of the tower is 100 m .
Soln.: Let the height of the tower be $B C$.
Horizontal distance $A B=100 \mathrm{~m}$
In $\triangle A B C, \tan \theta=\frac{B C}{A B}$
$\therefore \tan 60^{\circ}=\frac{B C}{100}$

$\Rightarrow \sqrt{3}=\frac{B C}{100} \Rightarrow B C=100 \sqrt{3} \mathrm{~m}$
Hence, height of the tower is $100 \sqrt{3} \mathrm{~m}$.
2. A vertical straight tree, 15 m high, is broken by the wind, in such a way that its top just touches the ground and makes an angle of $60^{\circ}$ with the ground. At what height from the ground did it break? (Use $\sqrt{3}=1.73$ )
Soln.: Let the height of the tree $A B=15 \mathrm{~m}$ It broke at $C$, its top $A$ touches the ground at $D$
Now, $A C=C D, \angle B D C=60^{\circ}$
Let $B C=h \mathrm{~m}$

$$
\begin{aligned}
& A C=A B-B C \\
& =(15-h) \mathrm{m} \\
\therefore \quad & A C=C D=15-h
\end{aligned}
$$

In $\triangle B C D, \sin 60^{\circ}=\frac{h}{15-h}$

$\Rightarrow \frac{\sqrt{3}}{2}=\frac{h}{15-h} \Rightarrow \sqrt{3}(15-h)=2 h$
$\Rightarrow 15 \sqrt{3}-\sqrt{3} h=2 h \Rightarrow 2 h+\sqrt{3} h=15 \sqrt{3}$
$\Rightarrow h(2+\sqrt{3})=15 \sqrt{3} \Rightarrow h=\frac{15 \sqrt{3}}{2+\sqrt{3}}$
$\Rightarrow h=\frac{15 \sqrt{3}(2-\sqrt{3})}{(2+\sqrt{3})(2-\sqrt{3})}$
$\Rightarrow h=\frac{30 \sqrt{3}-45}{4-3}=30 \times 1.73-45$

$$
=51.9-45=6.9 \mathrm{~m}
$$

Hence, the tree broke at the height of 6.9 m from the ground.
3. From the top of a 60 m high building, the angles of depression of the top and the bottom of a tower are observed to be $30^{\circ}$ and $60^{\circ}$ respectively. Find the height of the tower.
Soln.: Let $A B$ be the building and $C D$ be the tower. Let $C D=h$ metres. It is given that from the top of the building $B$, the angles of depression of the top $D$ and the bottom $C$ of the tower $C D$ are $30^{\circ}$ and $60^{\circ}$ respectively.
$\therefore \quad \angle E D B=30^{\circ}$ and $\angle A C B=60^{\circ}$
Let $A C=D E=x$
In $\triangle D E B$, right angled at $E$, we have
$\tan 30^{\circ}=\frac{B E}{D E}$
$\Rightarrow \frac{1}{\sqrt{3}}=\frac{60-h}{x}$

$\Rightarrow x=\sqrt{3}(60-h)$
In $\triangle C A B$, right-angled
at $A$, we have
$\tan 60^{\circ}=\frac{A B}{C A}$
$\Rightarrow \sqrt{3}=\frac{60}{x} \Rightarrow x=\frac{60}{\sqrt{3}}=\frac{60 \sqrt{3}}{3}=20 \sqrt{3}$
Putting the value of $x$ in (1), we get
$20 \sqrt{3}=\sqrt{3}(60-h)$
$\Rightarrow 20=60-h \Rightarrow h=60-20=40$ metres
Thus, the height of the tower is 40 metres.
4. The angle of elevation of a jet plane from a point $A$ on the ground is $60^{\circ}$. After a flight of 15 seconds, the angle of elevation changes to $30^{\circ}$. If the jet plane is flying at a constant height of $1500 \sqrt{3} \mathrm{~m}$, find the speed of the jet plane.
Soln.: Let $E$ be the position of the jet plane seen first time from $A$ and $C$ be the position of the jet plane seen after 15 seconds.

Let $A D=y \mathrm{~m}$ and $A B=x \mathrm{~m}$
$\angle E A D=60^{\circ}$ and $\angle C A B=30^{\circ}$


Height of the jet $=E D=B C=1500 \sqrt{3} \mathrm{~m}$
In $\triangle A B C, \tan 30^{\circ}=\frac{B C}{A B}$
$\Rightarrow \frac{1}{\sqrt{3}}=\frac{1500 \sqrt{3}}{x} \Rightarrow x=(1500 \sqrt{3}) \sqrt{3}=4500 \mathrm{~m}$
In $\triangle A D E, \tan 60^{\circ}=\frac{D E}{A D}$
$\Rightarrow \sqrt{3}=\frac{1500 \sqrt{3}}{y} \Rightarrow y=\frac{1500 \sqrt{3}}{\sqrt{3}}=1500 \mathrm{~m}$
$\therefore \quad$ Distance travelled in 15 seconds $=E C=D B$

$$
=x-y
$$

$$
=4500-1500=3000 \mathrm{~m}
$$

Speed $=\frac{\text { Distance }}{\text { Time }}=\frac{3000}{15} \mathrm{~m} / \mathrm{s}=200 \mathrm{~m} / \mathrm{s}$
Speed in km/hr. $=200 \times \frac{3600}{1000} \mathrm{~km} / \mathrm{hr}$

$$
=720 \mathrm{~km} / \mathrm{hr}
$$

5. Two ships are sailing in the sea on the either side of the light house, the angles of depression to two ships as observed from the top of the light house are $60^{\circ}$ and $45^{\circ}$, respectively. If the distance between the ships is $100\left(\frac{\sqrt{3}+1}{\sqrt{3}}\right) \mathrm{m}$, then find the height of the light house.
Soln.: Let $C D$ be the light house of the height $h \mathrm{~m}$ and let $A$ and $B$ be the two ships on either side of the light house, such that
$\angle C B D=45^{\circ}$ and $\angle C A D=60^{\circ}$
Given, $A B=\frac{100(\sqrt{3}+1)}{\sqrt{3}} \mathrm{~m}$
In $\triangle B D C, \angle D=90^{\circ}$
$\therefore \quad \frac{C D}{B D}=\tan 45^{\circ}$

$\Rightarrow \frac{C D}{B D}=1 \Rightarrow C D=B D$
Again, in $\triangle A D C, \angle D=90^{\circ}$
$\therefore \quad \frac{C D}{A D}=\tan 60^{\circ}$
$\Rightarrow \frac{C D}{A D}=\sqrt{3} \Rightarrow \frac{C D}{\sqrt{3}}=A D$
On adding (i) and (ii), we get
$A D+B D=\frac{C D}{\sqrt{3}}+C D$

$$
\begin{aligned}
& \Rightarrow \quad A B=C D\left(\frac{\sqrt{3}+1}{\sqrt{3}}\right) \\
& \Rightarrow 100\left(\frac{\sqrt{3}+1}{\sqrt{3}}\right)=C D\left(\frac{\sqrt{3}+1}{\sqrt{3}}\right) \\
& \quad\left(\because A B=\frac{100(\sqrt{3}+1)}{\sqrt{3}} \mathrm{~m}, \text { given }\right)
\end{aligned}
$$

On comparing, we get
$C D=100$
Hence, the height of light house is 100 m .
6. A person standing on the bank of a river, observes that the angle of elevation of the top of a tree, standing on the opposite bank is $60^{\circ}$. When he moves 40 m away from the bank, he finds the angle of elevation to be $30^{\circ}$. Find the height of the tree and width of the river.
Soln.: Let height of the tree $A B=y$ metre Width of the river $C B=x$ metre Let $C$ be the point of observation and $D$ be the other point of observation, such that
 $C D=40 \mathrm{~m}$
In $\triangle A B C$, right angled at $B$, we have

$$
\begin{align*}
& \tan 60^{\circ}=\frac{A B}{B C} \\
& \Rightarrow \sqrt{3}=\frac{y}{x} \Rightarrow \sqrt{3} x=y \Rightarrow y=\sqrt{3} x \tag{1}
\end{align*}
$$

In $\triangle A B D$, right angled at $B$, we have
$\tan 30^{\circ}=\frac{A B}{B D}$
$\Rightarrow \frac{1}{\sqrt{3}}=\frac{y}{x+40} \Rightarrow x+40=\sqrt{3} y$
From (1) and (2), we get
$x+40=\sqrt{3}(\sqrt{3} x)$
$\Rightarrow x+40=3 x \Rightarrow x=20$
Now, putting the value of $x$ in (1), we get

$$
y=20 \sqrt{3}=20(1.732)=34.64
$$

Hence, height of the tree $(y)=34.64$ metres and width of the river $(x)=20$ metres.
7. A vertical tower stands on a horizontal plane and is surmounted by a vertical flagstaff of height $h$. At a point on the plane, the angle of elevation of the bottom of the flagstaff is $\alpha$ and that of the top of flagstaff is $\beta$. Prove that the height of the tower is $\frac{h \boldsymbol{\operatorname { t a n } \alpha}}{\tan \beta-\boldsymbol{\operatorname { t a n }} \alpha}$.
Soln.: Let the height of the tower $B C$ be $x$ and $C D$ be the flagstaff. Let $C D=h$.
Let $A$ be the point of observation on the plane.
Let distance $A B=y, \angle B A C=\alpha$ and $\angle B A D=\beta$.
In $\triangle A B C$, right angled at $B$, we have
$\tan \alpha=\frac{B C}{A B}$
$\Rightarrow \tan \alpha=\frac{x}{y} \Rightarrow y=\frac{x}{\tan \alpha}$
In $\triangle A B D$, right angled at $B$, we have
$\tan \beta=\frac{B D}{A B}$
$\Rightarrow \tan \beta=\frac{h+x}{y} \Rightarrow y=\frac{h+x}{\tan \beta}$
From (1) and (2), we get
$\frac{x}{\tan \alpha}=\frac{h+x}{\tan \beta}$
$\Rightarrow \frac{x}{\tan \alpha}=\frac{h}{\tan \beta}+\frac{x}{\tan \beta}$
$\Rightarrow \frac{h}{\tan \beta}=\frac{x}{\tan \alpha}-\frac{x}{\tan \beta}$

$\Rightarrow \frac{h}{\tan \beta}=x\left(\frac{1}{\tan \alpha}-\frac{1}{\tan \beta}\right)$
$\Rightarrow \frac{h}{\tan \beta}=\frac{x(\tan \beta-\tan \alpha)}{\tan \alpha \tan \beta}$
$\Rightarrow x=\frac{h \tan \alpha \cdot \tan \beta}{\tan \beta(\tan \beta-\tan \alpha)}$
$\Rightarrow x=\frac{h \tan \alpha}{\tan \beta-\tan \alpha}$
Hence, height of the tower $=\frac{h \tan \alpha}{\tan \beta-\tan \alpha}$
8. From an aeroplane vertically above a straight horizontal plane, the angle of depression of two consecutive kilometre stones on the opposite sides of the aeroplane are found
to be $\alpha$ and $\beta$. Show that the height of the aeroplane is $\frac{\boldsymbol{\operatorname { t a n }} \alpha \boldsymbol{\operatorname { t a n }} \beta}{\boldsymbol{\operatorname { t a n }} \alpha+\boldsymbol{\operatorname { t a n }} \beta}$.
Soln.: Let $X Y$ be horizontal plane. Let the position of an aeroplane be at $A$.
Let $A B$ be the height of the aeroplane from the horizontal ground $X Y$.
So, $A B \perp X Y$
From $A$, the angle of depression of one kilometre stone $D$ is $\alpha$, i.e., $\angle A D B=\alpha$
Also, from $A$, the angle of depression of other kilometre stone $C$ is $\beta$, i.e., $\angle A C D=\beta$
Here $C D=1$ kilometre
$[\because$ Distance between two consecutive kilometre stones $C$ and $D$ is 1 km ]
So, $B C+B D=C D$
$\Rightarrow B C+B D=1 \mathrm{~km}$
Since $A B \perp X Y$ or $D C$, therefore

$$
\angle A B D=\angle A B C=90^{\circ}
$$

In the right $\triangle A B D$, we have
$\tan \alpha=\frac{A B}{B D}$

$\Rightarrow B D=\frac{A B}{\tan \alpha}$
In $\triangle A B C$, right angled at $B$, we have

$$
\begin{align*}
& \tan \beta=\frac{A B}{B C} \\
\Rightarrow & B C=\frac{A B}{\tan \beta} \tag{2}
\end{align*}
$$

Adding (1) and (2), we get
$B D+B C=\frac{A B}{\tan \alpha}+\frac{A B}{\tan \beta}[\because B D+B C=1 \mathrm{~km}]$

$$
\Rightarrow 1=A B\left(\frac{1}{\tan \alpha}+\frac{1}{\tan \beta}\right) \Rightarrow A B=\frac{\tan \alpha \tan \beta}{\tan \alpha+\tan \beta}
$$

Hence, the height of the aeroplane ( $A B$ )

$$
=\frac{\tan \alpha \tan \beta}{\tan \alpha+\tan \beta}
$$

9. The shadow of a tower standing on a leveled ground is found to be 40 m longer when the sun's altitude is $30^{\circ}$ than when it is $60^{\circ}$ ? Find the height of the tower.
Soln.: Let $A B$ be the tower and $A C$ and $A D$ be its shadows when the angles of elevation are $60^{\circ}$ and $30^{\circ}$. Then $C D=40$ metres. Let $h$ be
the height of the tower and let $A C=x$ metres.
In $\triangle A B C$,
right angled at $A$
$\tan 60^{\circ}=\frac{A B}{A C}$

$\Rightarrow \sqrt{3}=\frac{h}{x} \Rightarrow h=\sqrt{3} x \Rightarrow x=\frac{h}{\sqrt{3}}$
In $\triangle D A B$, we have
$\tan 30^{\circ}=\frac{A B}{A D}$
$\Rightarrow \frac{1}{\sqrt{3}}=\frac{h}{x+40} \Rightarrow x+40=\sqrt{3} h$
Putting value of $x$ from (1) in (2), we get

$$
\begin{aligned}
& \frac{h}{\sqrt{3}}+40=\sqrt{3} h \\
\Rightarrow & h+40 \sqrt{3}=3 h \Rightarrow 2 h=40 \sqrt{3} \Rightarrow h=20 \sqrt{3}
\end{aligned}
$$

Thus, the height of the tower is $20 \sqrt{3}$ metres.
10. A round balloon of radius $r$ subtends an angle $\alpha$ at the eye of the observer while the angle of elevation of its centre is $\beta$. Prove that the height of the centre of the balloon is $r \sin \beta \operatorname{cosec} \alpha / 2$.
Soln.: Let $O$ be the centre of the balloon of radius $r$ and $P$ be the eye of the observer. Let $P A$, $P B$ be tangents from $P$ to the balloon. Then $\angle A P B=\alpha$
$\therefore \quad \angle A P O=\angle B P O=\alpha / 2$
Let $O Q$ be perpendicular from $O$ on the horizontal $P X$. We are given that the angle of the elevation of the centre of the balloon is $\beta$. i.e., $\angle O P Q=\beta$
In $\triangle O P B$, right angled at $B$,

$$
\begin{align*}
& \sin \frac{\alpha}{2}=\frac{O B}{O P} \\
& \Rightarrow \sin \frac{\alpha}{2}=\frac{r}{O P} \\
& \Rightarrow O P=\frac{r}{\sin \frac{\alpha}{2}} \tag{1}
\end{align*}
$$


$\Rightarrow O P=r \operatorname{cosec} \frac{\alpha}{2}$
Now, in $\triangle O P Q$, right angled at $Q$,

$$
\begin{align*}
& \sin \beta=\frac{O Q}{O P} \\
\Rightarrow & O Q=O P \sin \beta \tag{2}
\end{align*}
$$

Putting the value of $O P$ from (1) in (2), we get

$$
O Q=r \operatorname{cosec} \alpha / 2 \sin \beta
$$

Hence, the height of the centre of the balloon is $r \sin \beta \operatorname{cosec} \alpha / 2$
11. There is a small island in the middle of a 100 m wide river and a tall tree stands on the island. $P$ and $Q$ are points directly opposite to each other on two banks and in line with the tree. If the angles of elevation of the top of the tree from $P$ and $Q$ are respectively $30^{\circ}$ and $45^{\circ}$, then find the height of the tree.
Soln.: Let $O A$ be the tree of height $h \mathrm{~m}$.
Given, $P Q=100 \mathrm{~m}$
In $\triangle P O A$, we have
$\tan 30^{\circ}=\frac{O A}{O P}$

$\Rightarrow \frac{1}{\sqrt{3}}=\frac{h}{O P} \Rightarrow O P=\sqrt{3} h$
Now, in $\triangle Q O A$,
$\tan 45^{\circ}=\frac{O A}{O Q}$
$\Rightarrow 1=\frac{h}{O Q} \Rightarrow O Q=h$
On adding equation (1) and (2), we get
$O P+O Q=\sqrt{3} h+h$
$\Rightarrow P Q=(\sqrt{3}+1) h$
$\Rightarrow 100=(\sqrt{3}+1) h \quad(\because P Q=100 \mathrm{~m})$
$\Rightarrow h=\frac{100}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} \quad$ (by rationalisation)
$\Rightarrow h=\frac{100(\sqrt{3}-1)}{2}$
$\Rightarrow h=50(\sqrt{3}-1)=50(1.732-1)$
$\Rightarrow h=36.6 \mathrm{~m}$
Hence, height of the tree of 36.6 m .
12. A ladder rests against a wall at an angle $\alpha$ to the horizontal. Its foot is pulled away from the wall through a distance $a$, so that it slides a distance $b$ down the wall making an angle $\beta$ with the horizontal, show that $\frac{a}{b}=\frac{\cos \alpha-\cos \beta}{\sin \beta-\sin \alpha}$.
Soln.: Let the ladder be $A C$ making an angle $\alpha$ with the horizontal. On pulling, the ladder comes
in the position $A^{\prime} C^{\prime}$ making an angle $\beta$ with the horizontal.
$A^{\prime} A=a, C^{\prime} C=b, A C=A^{\prime} C^{\prime}=h=$ length of ladder,
$B C^{\prime}=x, A B=y$
In $\triangle A B C$, right angled at $B$

$$
\begin{align*}
\sin \alpha & =\frac{B C}{A C} \\
\Rightarrow \sin \alpha & =\frac{x+b}{h} \tag{2}
\end{align*}
$$


and $\cos \alpha=\frac{A B}{A C} \Rightarrow \cos \alpha=\frac{y}{h}$
Now, in $\Delta A^{\prime} B C^{\prime}$, right angled at $B$
$\sin \beta=\frac{B C^{\prime}}{A^{\prime} C^{\prime}}$
$\Rightarrow \quad \sin \beta=\frac{x}{h}$
and $\cos \beta=\frac{A^{\prime} B}{A^{\prime} C^{\prime}} \Rightarrow \cos \beta=\frac{a+y}{h}$
Now, $\frac{\cos \alpha-\cos \beta}{\sin \beta-\sin \alpha}=\frac{\frac{y}{h}-\frac{(a+y)}{h}}{\frac{x}{h}-\frac{x+b}{h}}$
$\Rightarrow \frac{\cos \alpha-\cos \beta}{\sin \beta-\sin \alpha}=\frac{\frac{y-a-y}{h}}{\frac{x-x-b}{h}}$
$\Rightarrow \frac{\cos \alpha-\cos \beta}{\sin \beta-\sin \alpha}=\frac{-a}{-b}=\frac{a}{b}$
Hence, $\frac{a}{b}=\frac{\cos \alpha-\cos \beta}{\sin \beta-\sin \alpha}$
13. The angle of elevation of a cliff from a fixed point is $\theta$. After going up a distance $k$ metres towards the top of the cliff at an angle of $\phi$, it is found that the angle of elevation is $\alpha$, show that the height of the cliff is $\frac{k(\cos \phi-\sin \phi \cot \alpha)}{\cot \theta-\cot \alpha}$.
Soln.: Let $A B$ be the cliff and $O$ be the fixed point such that the angle of elevation of the cliff from $O$ is $\theta$, i.e., $\angle A O B=\theta$. Let $\angle E O C=\phi$ and $O C=k$ metres. From $C$ draw $C D$ and $C E$ perpendiculars on $A B$ and $O A$ respectively. Then, $\angle D C B=\alpha$, let $h$ be the height of the cliff $A B$.


In $\triangle O C E$, we have
$\sin \phi=\frac{C E}{O C} \Rightarrow \sin \phi=\frac{C E}{k} \Rightarrow C E=k \sin \phi$
and $\cos \phi=\frac{O E}{O C} \Rightarrow \cos \phi=\frac{O E}{k} \Rightarrow O E=k \cos \phi$
In $\triangle O A B$, we have
$\tan \theta=\frac{A B}{O A}$
$\Rightarrow \tan \theta=\frac{h}{O A} \Rightarrow O A=\frac{h}{\tan \theta}$
$\Rightarrow O A=h \cot \theta \Rightarrow O E+E A=h \cot \theta$
$\Rightarrow E A=h \cot \theta-O E=h \cot \theta-k \cos \phi$

$$
[\because O E=k \cos \phi]
$$

In $\triangle B C D$, we have

$$
\begin{gathered}
\cot \alpha=\frac{C D}{B D} \Rightarrow \cot \alpha=\frac{A E}{A B-A D}[\because C D=A E] \\
\Rightarrow \quad \cot \alpha=\frac{h \cot \theta-k \cos \phi}{h-k \sin \phi} \\
\quad[\because A D=C E=k \sin \phi, O E=k \cos \phi] \\
\Rightarrow h \cot \alpha-k \sin \phi \cot \alpha=h \cot \theta-k \cos \phi \\
\Rightarrow h \cot \theta-h \cot \alpha=k \cos \phi-k \sin \phi \cot \alpha \\
\Rightarrow h(\cot \theta-\cot \alpha)=k(\cos \phi-\sin \phi \cot \alpha) \\
\Rightarrow h=\frac{k(\cos \phi-\sin \phi \cot \alpha)}{\cot \theta-\cot \alpha}
\end{gathered}
$$

14. A boy is standing on the ground and flying a kite with a string of length 150 m at an angle of elevation of $30^{\circ}$, another boy is standing on the roof of a 25 m high building and is flying his kite at an elevation of $45^{\circ}$. Both the boys are on opposite sides of both the kites. Find the length of the string (in metres) corrects to two decimal places, that the second boy must have so that the two kites meet.
Soln.: Let $A$ be the position of first boy and $D$ be the position of kite. Let $Q D$ be the vertical height of the kite.

$A D$ is the string i.e., $A D=150 \mathrm{~m}, \angle D A Q=30^{\circ}$ Now in $\triangle A Q D$, right angled at $Q$,

$$
\sin 30^{\circ}=\frac{D Q}{A D}
$$

$$
\Rightarrow \frac{1}{2}=\frac{D Q}{150} \Rightarrow D Q=\frac{150}{2}=75 \mathrm{~m}
$$

Let $B$ be the position of second boy and $B P$ be the roof. So, $P Q=B C=25 \mathrm{~m}$
Then second kite meets the first kite at $D$. Then $\angle D B P=45^{\circ}$
and $D P=D Q-P Q=75-25=50 \mathrm{~m}$

$$
[\because D Q=75 \mathrm{~m}]
$$

In $\triangle D P B$, right angled at $P$

$$
\begin{aligned}
& \sin 45^{\circ}=\frac{D P}{D B} \\
& \Rightarrow \frac{1}{\sqrt{2}}=\frac{50}{D B} \\
& \Rightarrow D B=50 \sqrt{2}=50(1.4142)=70.71
\end{aligned}
$$

Hence, length of the string $=70.71 \mathrm{~m}$
15. From a window ( $h$ metres high above the ground) of a house in a street, the angles of elevation and depression of the top and the foot of another house on the opposite side of the street are $\theta$ and $\phi$ respectively. Show that the height of the opposite house is $h(1+\tan \theta \cot \phi)$
Soln.: Let $W$ be the window and $A B$ be the house on the opposite side. Then WP is the width of the street, height of the window $=h$ metres $=B P$. Let $A P=x$ metres and $W P=y$ metres In $\triangle B P W$, right angled at $P$,

$$
\begin{aligned}
& \tan \phi=\frac{B P}{W P} \\
\Rightarrow & \tan \phi=\frac{h}{y} \\
\Rightarrow & y=\frac{h}{\tan \phi}=h \cot \phi
\end{aligned}
$$



Now, in $\triangle \mathrm{APW}$, we have

$$
\begin{aligned}
& \tan \theta=\frac{A P}{W P} \\
\Rightarrow & \tan \theta=\frac{x}{y} \Rightarrow x=y \tan \theta \\
\Rightarrow & x=h \cot \phi \tan \theta \quad[\because y=h \cot \phi]
\end{aligned}
$$

Height of the opposite house $=A P+B P$
$=x+h=h \cot \phi \tan \theta+h=h(\cot \phi \tan \theta+1)$
$=h(1+\tan \theta \cot \phi)$.

## NCERT SECTION

## Exercise 9.1

1. A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole, if the angle by the rope with the ground level is $30^{\circ}$ (see figure).


Soln.: In right $\triangle A B C$,
$\frac{A B}{A C}=\sin 30^{\circ}$
$\Rightarrow \frac{A B}{A C}=\frac{1}{2} \Rightarrow \frac{A B}{20}=\frac{1}{2}$

$$
\left[\because A C=20 \mathrm{~m} \text { and } \sin 30^{\circ}=\frac{1}{2}\right]
$$

$\Rightarrow A B=20 \times \frac{1}{2}=10 \mathrm{~m}$
Thus, the required height of the pole is 10 m .
2. A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle $30^{\circ}$ with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m . Find the height of the tree.
Soln.: Let tree is broken at $A$ and its top is touching the ground at $B$.
Now, in right $\triangle A O B$, we have
$\frac{A O}{O B}=\tan 30^{\circ}$
$\Rightarrow \quad \frac{A O}{O B}=\frac{1}{\sqrt{3}}$

$\Rightarrow \frac{A O}{8}=\frac{1}{\sqrt{3}} \Rightarrow A O=\frac{8}{\sqrt{3}} \mathrm{~m}$
Also, $\frac{A B}{O B}=\sec 30^{\circ}$
$\Rightarrow \frac{A B}{8}=\frac{2}{\sqrt{3}} \Rightarrow A B=\frac{2 \times 8}{\sqrt{3}}=\frac{16}{\sqrt{3}} \mathrm{~m}$
Now, height of the tree $O P=O A+A B$
$=\frac{8}{\sqrt{3}}+\frac{16}{\sqrt{3}}=\frac{24}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \mathrm{~m}=8 \sqrt{3} \mathrm{~m}$
3. A contractor plans to install two slides for the children to play in a park. For the children below the age of 5 years, she prefers to have a slide whose top is at a height of 1.5 m , and is inclined at an angle of $30^{\circ}$ to the ground, whereas for elder children, she wants to have a steep slide at a height of 3 m , and inclined at angle of $60^{\circ}$ to the ground. What should be the length of the slide in each case?
Soln.: In the figure, $D E$ is the slide for younger children whereas $A C$ is the slide for elder children.


In right $\triangle A B C, A B=3 \mathrm{~m}$

$$
\begin{aligned}
& \therefore \frac{A B}{A C}=\sin 60^{\circ} \\
& \Rightarrow \frac{3}{A C}=\frac{\sqrt{3}}{2} \Rightarrow A C=\frac{2 \times 3}{\sqrt{3}}=2 \sqrt{3} \mathrm{~m}
\end{aligned}
$$

Again in right $\triangle B D E$,

$$
\begin{gathered}
\frac{D E}{B D}=\operatorname{cosec} 30^{\circ}=2 \\
\Rightarrow \frac{D E}{1.5}=2 \Rightarrow D E=2 \times 1.5 \mathrm{~m} \Rightarrow D E=3 \mathrm{~m}
\end{gathered}
$$

Thus, the lengths of slides are 3 m and $2 \sqrt{3} \mathrm{~m}$.
4. The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of the tower, is $30^{\circ}$. Find the height of the tower.
Soln.: In right $\triangle A B C, A B=$ height of the tower and point $C$ is 30 m away from the foot of the tower,
$\therefore \quad A C=30 \mathrm{~m}$
Now, $\frac{A B}{A C}=\tan 30^{\circ}$
$\Rightarrow \frac{h}{30}=\frac{1}{\sqrt{3}}$

$\Rightarrow h=\frac{30}{\sqrt{3}}=\frac{30}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}=10 \sqrt{3}$
Thus, the required height of the tower is $10 \sqrt{3} \mathrm{~m}$.
5. A kite is flying at a height of $\mathbf{6 0} \mathbf{m}$ above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is $60^{\circ}$. Find the length of the string, assuming that there is no slack in the string.
Soln.: Let $O B=$ Length of the string $A B=60 \mathrm{~m}=$ Height of the kite. In the right $\triangle A O B$,
$\therefore \quad \frac{O B}{A B}=\operatorname{cosec} 60^{\circ}=\frac{2}{\sqrt{3}}$


$$
\begin{aligned}
& \Rightarrow \frac{O B}{60}=\frac{2}{\sqrt{3}} \\
& \Rightarrow O B=\frac{2 \times 60}{\sqrt{3}}=\frac{120 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}=40 \sqrt{3}
\end{aligned}
$$

Thus, length of the string is $40 \sqrt{3}$.
6. A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from $30^{\circ}$ to $60^{\circ}$ as he walks towards the building. Find the distance he walked towards the building.
Soln.: Here, $O A$ is the building.


In right $\triangle A B D$,

$$
\begin{aligned}
& \frac{A D}{B D}=\tan 30^{\circ}=\frac{1}{\sqrt{3}} \\
& \Rightarrow B D=A D \sqrt{3}=28.5 \sqrt{3} \\
& \quad[\because A D=30 \mathrm{~m}-1.5 \mathrm{~m}=28.5 \mathrm{~m}]
\end{aligned}
$$

Also, in right $\triangle A C D$,
$\frac{A D}{C D}=\tan 60^{\circ}=\sqrt{3} \Rightarrow C D=\frac{A D}{\sqrt{3}}=\frac{28.5}{\sqrt{3}}$
Now, $\frac{A D}{C D}=\tan 60^{\circ}=\sqrt{3} \Rightarrow C D=\frac{A D}{\sqrt{3}}=\frac{28.5}{\sqrt{3}}$
Now, $B C=B D-C D=28.5 \sqrt{3}-\frac{28.5}{\sqrt{3}}$

$$
\Rightarrow B C=28.5\left[\sqrt{3}-\frac{1}{\sqrt{3}}\right]=28.5\left[\frac{3-1}{\sqrt{3}}\right]
$$

$=28.5 \times \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}=\frac{28.5 \times 2 \times \sqrt{3}}{3}$
$=9.5 \times 2 \times \sqrt{3}=19 \sqrt{3} \mathrm{~m}$
Thus the distance walked by the boy towards the building is $19 \sqrt{3} \mathrm{~m}$.
7. From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are $45^{\circ}$ and $60^{\circ}$ respectively. Find the height of the tower.
Soln.: Let the height of the building be $B C$
$\therefore B C=20 \mathrm{~m}$
and height of the tower be $C D$.
Let the point $A$ be at a distance $y$ metres from the foot of the building.
Now, in right $\triangle A B C$,
$\frac{B C}{A B}=\tan 45^{\circ}=1$

$\Rightarrow \frac{20}{y}=1 \Rightarrow y=20 \mathrm{~m}$ i.e., $A B=20 \mathrm{~m}$.
Now, in right $\triangle A B D$,
$\frac{B D}{A B}=\tan 60^{\circ}=\sqrt{3}$
$\Rightarrow \frac{B D}{20}=\sqrt{3}$
$\Rightarrow \frac{20+x}{20}=\sqrt{3} \Rightarrow 20+x=20 \sqrt{3}$
$\Rightarrow x=20 \sqrt{3}-20=20[\sqrt{3}-1]$
$\Rightarrow x=20 \times 0.732=14.64$
Thus, the height of the tower is 14.64 m .
8. A statue, 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is $60^{\circ}$ and from the same point the angle of elevation of the top of the pedestal is $45^{\circ}$. Find the height of the pedestal.
Soln.: In the figure, $D C$ represents the statue and $B C$ represents the pedestal.
Now, in right $\triangle A B C$, we have
$\frac{A B}{B C}=\cot 45^{\circ}=1$
$\Rightarrow \frac{A B}{h}=1 \Rightarrow A B=h$ metres.
Now in right $\triangle A B D$, we have
$\frac{B D}{A B}=\tan 60^{\circ}=\sqrt{3}$

$\Rightarrow B D=\sqrt{3} \times A B=\sqrt{3} \times h$
$\Rightarrow h+1.6=\sqrt{3} h \Rightarrow h(\sqrt{3}-1)=1.6$
$\Rightarrow h=\frac{1.6}{\sqrt{3}-1}=\frac{1.6}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$
$\Rightarrow h=\frac{1.6}{3-1} \times(\sqrt{3}+1)=\frac{1.6}{2} \times \sqrt{3}+1$

$$
=0.8(\sqrt{3}+1) \mathrm{m}
$$

Thus, the height of the pedestal is

$$
0.8(\sqrt{3}+1) \mathrm{m}
$$

9. The angle of elevation of the top of a building from the foot of the tower is $30^{\circ}$ and the angle of elevation of the top of the tower from the foot of the building is $60^{\circ}$. If the tower is 50 m high, find the height of the building. Soln.: In the figure, let height of the building $=A B$

$$
=h \mathrm{~m}
$$



Let $C D$ be the tower.
$\therefore C D=50 \mathrm{~m}$
Now, in right $\triangle A B C$,

$$
\begin{align*}
& \frac{A C}{A B}=\cot 30^{\circ}=\sqrt{3} \\
\Rightarrow & \frac{A C}{h}=\sqrt{3} \Rightarrow A C=h \sqrt{3} \tag{1}
\end{align*}
$$

In right $\triangle D C A$,

$$
\begin{equation*}
\frac{D C}{A C}=\tan 60^{\circ} \tag{2}
\end{equation*}
$$

$\Rightarrow \frac{50}{A C}=\sqrt{3} \Rightarrow A C=\frac{50}{\sqrt{3}}$
From (1) and (2), we get
$\sqrt{3} h=\frac{50}{\sqrt{3}} \Rightarrow h=\frac{50}{\sqrt{3}} \times \frac{1}{\sqrt{3}}=\frac{50}{3}$
Thus, the height of the building $=16 \frac{2}{3} \mathrm{~m}$.
10. Two poles of equal heights are standing opposite each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are $60^{\circ}$ and $30^{\circ}$ respectively. Find the height of the poles and the distances of the point from the poles.

Soln.: $A B=h$ metres
$C D=h$ metres
$A P=x \mathrm{~m}$
$C P=(80-x) \mathrm{m}$
Now, in right $\triangle A P B$, we have

$\frac{A B}{A P}=\tan 60^{\circ}$
$\Rightarrow \frac{h}{x}=\sqrt{3} \Rightarrow h=x \sqrt{3}$


Again in right $\triangle C P D$, we have

$$
\frac{C D}{C P}=\tan 30^{\circ}
$$

$$
\begin{equation*}
\Rightarrow \frac{h}{(80-x)}=\frac{1}{\sqrt{3}} \Rightarrow h=\frac{80-x}{\sqrt{3}} \tag{2}
\end{equation*}
$$

From (1) and (2), we get
$\sqrt{3} x=\frac{80-x}{\sqrt{3}}$
$\Rightarrow \sqrt{3} \times \sqrt{3} \times x=80-x \Rightarrow 3 x=80-x$
$\Rightarrow 3 x+x=80 \Rightarrow 4 x=80 \Rightarrow x=\frac{80}{4}=20$
$\therefore C P=80-x=80-20=60 \mathrm{~m}$
Now, from (1), we have
$h=\sqrt{3} \times 20=1.732 \times 20=34.64$
Thus, the required point is 20 m away from the first pole and 60 m away from the second pole.
Height of each pole $=34.64 \mathrm{~m}$.
11. A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is $60^{\circ}$. From another point 20 m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of tower is $30^{\circ}$ (see figure). Find the height of the tower and the width of the canal.


Soln.: Let the TV tower be $A B=h \mathrm{~m}$.
Let the point ' $C$ ' be such that $B C=x$ and $C D=20 \mathrm{~m}$.
Now, in right $\triangle A B C$, we have
$\frac{A B}{B C}=\tan 60^{\circ}$
$\Rightarrow \frac{h}{x}=\sqrt{3} \Rightarrow h=\sqrt{3} x$
In right $\triangle A B D$, we have

$$
\begin{align*}
& \frac{A B}{B D}=\tan 30^{\circ} \\
\Rightarrow & \frac{h}{x+20}=\frac{1}{\sqrt{3}} \Rightarrow h=\frac{x+20}{\sqrt{3}} \tag{2}
\end{align*}
$$

From (1) and (2), we get
$\sqrt{3} x=\frac{x+20}{\sqrt{3}} \Rightarrow 3 x=x+20$
$\Rightarrow 3 x-x=20 \Rightarrow 2 x=20 \Rightarrow x=\frac{20}{2}=10 \mathrm{~m}$
Now, from (1), we get
$h=\sqrt{3} \times 10=1.732 \times 10=17.32$
Thus, the height of the tower $=17.32 \mathrm{~m}$.
Also width of the canal $=10 \mathrm{~m}$.
12. From the top of a 7 m high building, the angle of elevation of the top of a cable tower is $60^{\circ}$ and the angle of depression of its foot is $45^{\circ}$. Determine the height of the tower.
Soln.: In the figure, let $A B$ be the height of the building.
$\therefore A B=7$ metres.


Let $C D$ be the cable tower.
$\therefore$ In right $\triangle D A E$, we have

$$
\begin{align*}
& \frac{D E}{E A}=\tan 60^{\circ} \Rightarrow \frac{h}{x}=\sqrt{3} \\
& \Rightarrow h=\sqrt{3} \cdot x \tag{1}
\end{align*}
$$

Again, in right $\triangle A B C$, we have

$$
\frac{A B}{B C}=\tan 45^{\circ}
$$

$$
\begin{equation*}
\Rightarrow \frac{7}{x}=1 \Rightarrow x=7 \tag{2}
\end{equation*}
$$

From (1) and (2), we get

$$
\begin{aligned}
h= & 7 \sqrt{3}=D E \\
\therefore \quad C D & =C E+E D \\
& =7+7 \sqrt{3}=7(1+\sqrt{3}) \mathrm{m} \\
& =7(1+1.732) \mathrm{m}=7(2.732) \mathrm{m}=19.124 \mathrm{~m}
\end{aligned}
$$

13. As observed from the top of a 75 m high light house from the sea-level, the angles of depression of two ships are $30^{\circ}$ and $45^{\circ}$. If one ship is exactly behind the other on the same side of the light house, find the distance between the two ships.
Soln.: In the figure, let $A B$ represent the light house. $\therefore A B=75 \mathrm{~m}$.
Let the two ships be $C$ and $D$ such that angle of depression from $A$ are $45^{\circ}$ and $30^{\circ}$ respectively.
Now, in right $\triangle A B C$, we have
$\frac{A B}{B C}=\tan 45^{\circ}$

$\Rightarrow \frac{75}{B C}=1 \Rightarrow B C=75$
Again, in right $\triangle A B D$, we have
$\frac{A B}{B D}=\tan 30^{\circ} \Rightarrow \frac{75}{B D}=\frac{1}{\sqrt{3}} \Rightarrow B D=75 \sqrt{3}$
Since the distance between the two ships $=C D$

$$
\begin{aligned}
& =B D-B C=75 \sqrt{3}-75=75[\sqrt{3}-1] \\
& \quad=75[1.732-1]=75 \times 0.732=54.9
\end{aligned}
$$

Thus, the required distance between the ships is 54.9 m .
14. A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is $60^{\circ}$. After some time, the angle of elevation reduces to $30^{\circ}$ (see figure). Find the distance travelled by the balloon during the interval.


Soln.: In the figure, let $C$ be the position of the observer (the girl).
$A$ and $P$ are two positions of the balloon. $C D$ is the horizontal line from the eyes of the observer (girl).
Here $P D=A B=88.2 \mathrm{~m}-1.2 \mathrm{~m}=87 \mathrm{~m}$
In right $\triangle A B C$, we have
$\frac{A B}{B C}=\tan 60^{\circ}$
$\Rightarrow \frac{87}{B C}=\sqrt{3} \Rightarrow B C=\frac{87}{\sqrt{3}} \mathrm{~m}$
In right $\triangle P D C$, we have

$$
\begin{aligned}
& \frac{P D}{C D}=\tan 30^{\circ} \\
\Rightarrow & \frac{87}{C D}=\frac{1}{\sqrt{3}} \Rightarrow C D=87 \sqrt{3}
\end{aligned}
$$

Now, $B D=C D-B C$
$=87 \sqrt{3}-\frac{87}{\sqrt{3}}=87\left[\sqrt{3}-\frac{1}{\sqrt{3}}\right]$
$=87 \times\left(\frac{3-1}{\sqrt{3}}\right)=\frac{2 \times 87}{\sqrt{3}} \mathrm{~m}$
$=\frac{2 \times 87}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}=\frac{2 \times 87 \times \sqrt{3}}{3}=2 \times 29 \times \sqrt{3} \mathrm{~m}$
$=58 \sqrt{3} \mathrm{~m}$
Thus, the required distance between the two positions of the balloon $=58 \sqrt{3} \mathrm{~m}$.
15. A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at angle of depression of $30^{\circ}$, which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be $60^{\circ}$. Find the time taken by the car to reach the foot of the tower from this point.
Soln.: In the figure, let $A B$ be the height of the tower and $C, D$ be the two positions of the car.


In right $\triangle A B D$, we have
$\frac{A B}{A D}=\tan 60^{\circ}$
$\Rightarrow \frac{A B}{A D}=\sqrt{3} \Rightarrow A B=\sqrt{3} \cdot A D$
In right $\triangle A B C$, we have
$\frac{A B}{A C}=\tan 30^{\circ}$
$\Rightarrow \frac{A B}{A C}=\frac{1}{\sqrt{3}} \Rightarrow A B=\frac{A C}{\sqrt{3}}$

From (1) and (2), we get
$\sqrt{3} A D=\frac{A C}{\sqrt{3}}$
$\Rightarrow A C=\sqrt{3} \times \sqrt{3} \times A D=3 A D$
Now, $C D=A C-A D=3 A D-A D=2 A D$
Since the distance $2 A D$ is covered in 6 seconds,
$\therefore$ The distance $A D$ will be covered in $\frac{6}{2}$ i.e., 3 seconds,
Thus, the time taken by the car to reach the tower from $D$ is 3 seconds.
16. The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is $6 \mathbf{m}$.
Soln.: Let the tower be represented by $A B$ in the figure.


Let $A B=h$ metres.
$\therefore$ In right $\triangle A B C$, we have
$\frac{A B}{A C}=\tan \theta$
$\Rightarrow \frac{h}{9}=\tan \theta$
In right $\triangle A B D$, we have
$\frac{A B}{A D}=\tan \left(90^{\circ}-\theta\right)=\cot \theta$
$\Rightarrow \frac{h}{4}=\cot \theta$
Multiplying (1) and (2), we get
$\frac{h}{9} \times \frac{h}{4}=\tan \theta \times \cot \theta=1 \quad[\because \tan \theta \times \cot \theta=1]$
$\Rightarrow \frac{h^{2}}{36}=1 \Rightarrow h^{2}=36$
$\Rightarrow h= \pm 6 \mathrm{~m} \quad \therefore h=6 \mathrm{~m}$
[ $\because$ Height is positive only]
Thus, the height of the tower is 6 m .

## EXERCISE

## Multiple Choice Questions

## Level - 1

1. The angle of depression of the top and bottom of a 7 m tall building from the top of a tower are $45^{\circ}$ and $60^{\circ}$ respectively. The height of the tower is
(a) 16.56 m
(b) 16.06 m
(c) 16.50 m
(d) 16.68 m
2. The angle of elevation of the sun (sun's altitude) when the length of a shadow of a vertical pole is equal to its height is
(a) $30^{\circ}$
(b) $45^{\circ}$
(c) $60^{\circ}$
(d) $90^{\circ}$
3. A tree breaks due to the storm and the broken part bends such that the top of the tree touches the ground making an angle of $30^{\circ}$ with the ground. The distance from the foot of the tree to the point where the top touches the ground is 10 m . The height of the tree is
(a) $10 \sqrt{3} \mathrm{~m}$
(b) $10(\sqrt{3}+1) \mathrm{m}$
(c) $10(\sqrt{3}-1) \mathrm{m}$
(d) $\frac{10}{\sqrt{3}} \mathrm{~m}$
4. The length of a string between a kite and a point on the ground is 85 m . If the string makes an angle $\theta$ with level ground such that $\tan \theta=\frac{15}{8}$, how high is the kite?
(a) 75 m
(b) 78.05 m
(c) 226 m
(d) None of these
5. The angle of elevation of the top of a tower from a distance of 100 m from its foot is $30^{\circ}$. The height of the tower is
(a) $100 \sqrt{3}$
(b) $\frac{100}{\sqrt{3}} \mathrm{~m}$
(c) $50 \sqrt{3}$
(d) $\frac{200}{\sqrt{3}} \mathrm{~m}$
6. A person walking 20 m towards a chimney in a horizontal line through its base observes that its angle of elevation changes from $30^{\circ}$ to $45^{\circ}$.

The height of chimney is
(a) $\frac{20}{\sqrt{3}+1} \mathrm{~m}$
(b) $\frac{20}{\sqrt{3}-1} \mathrm{~m}$
(c) $20(\sqrt{3}-1) \mathrm{m}$
(d) None of these
7. The angle of elevation of a ladder leaning against a wall is $60^{\circ}$ and the foot of the ladder is 9.5 m away from the wall. The length of the ladder is
(a) 10 m
(b) 16 m
(c) 18 m
(d) 19 m
8. The ratio of the length of a rod and its shadow is $1: \sqrt{3}$. The altitude of the sun is
(a) $30^{\circ}$
(b) $45^{\circ}$
(c) $60^{\circ}$
(d) $90^{\circ}$
9. The angle of elevation of the top of a tower standing on a horizontal plane from a point $A$ is $\alpha$. After walking a distance $d$ towards the foot of the tower the angle of elevation is found to be $\beta$. The height of the tower is
(a) $\frac{d}{\cot \alpha+\cot \beta}$
(b) $\frac{d}{\cot \alpha-\cot \beta}$
(c) $\frac{d}{\tan \beta-\tan \alpha}$
(d) $\frac{d}{\tan \beta+\tan \alpha}$
10. The top of two poles of height 20 m and 14 m are connected by a wire. If the wire makes an angle of $30^{\circ}$ with the horizontal, then the length of the wire is
(a) 12 m
(b) 10 m
(c) 8 m
(d) 6 m
11. Two men standing on opposite sides of a flagstaff measure the angles of the top of the flagstaff is $30^{\circ}$ and $60^{\circ}$. If the height of the flagstaff is 20 m , distance between the men is
(a) 46.19 m
(b) 40 m
(c) 50 m
(d) 30 m
12. The angles of elevation of the top of a tower from the points $C$ and $D$ at distance of $a$ and $b$ respectively from the base and in the same straight line with it are complementary. The
height of the tower is
(a) $a b$
(b) $\sqrt{a b}$
(c) $\sqrt{\frac{a}{b}}$
(d) $\sqrt{\frac{b}{a}}$
13. The angles of elevation of an artificial satellite measured from two earth stations are $30^{\circ}$ and $60^{\circ}$ respectively. If the distance between the earth stations is 4000 km , then the height of the satellite is
(a) 2000 km
(b) 6000 km
(c) 3464 km
(d) 2828 km
14. One side of a parallelogram is 12 cm and its area is $60 \mathrm{~cm}^{2}$. If the angle between the adjacent side is $30^{\circ}$, then its other sides is
(a) 8 cm
(b) 6 cm
(c) 10 cm
(d) 4 cm
15. A tree 6 m tall cast a 4 m long shadow. At the same time, a flag pole casts a shadow 50 m long. How long is the flag pole?
(a) 75 m
(b) 100 m
(c) 150 m
(d) 50 m

## Level - 2

16. Two persons are ' $a$ ' metres apart and the height of one is double that of the other. If from the middle point of the line joining their feet, an observer finds the angular elevations of their tops to be complementary, then the height of the shortest persons in metres is
(a) $\frac{a}{4}$
(b) $\frac{a}{\sqrt{2}}$
(c) $a \sqrt{2}$
(d) $\frac{a}{2 \sqrt{2}}$
17. If the angle of elevation of a cloud from a point 200 m above a lake is $30^{\circ}$ and the angle of depression of its reflection in the lake is $60^{\circ}$, then the height of the cloud above the lake is
(a) 200 m
(b) 500 m
(c) 30 m
(d) 400 m
18. The angle of elevation of a cloud from a point $h$ metre above a lake is $\theta$. The angle of depression of its reflection in the lake is $45^{\circ}$. The height of the cloud is
(a) $h \tan \left(45^{\circ}-\theta\right)$
(b) $h \cot \left(45^{\circ}+\theta\right)$
(c) $h \tan \left(45^{\circ}+\theta\right)$
(d) $h \cot \left(45^{\circ}-\theta\right)$
19. The angle of elevation of the top of a rock from the top and foot of 100 m high tower are $30^{\circ}$ and $45^{\circ}$ respectively. The height of the rock is
(a) $50(3-\sqrt{3}) \mathrm{m}$
(b) $50(3+\sqrt{3}) \mathrm{m}$
(c) $50 \sqrt{3} \mathrm{~m}$
(d) 150 m
20. Mr. Anna Hazare, Padyatra Party wanted to go from Delhi to Dehradun. The Walkers travelled 150 km straight and then took a $45^{\circ}$ turn towards Varanasi and walked straight for another 120 km . Approximately how far was the party from the starting point?
(a) 192 km
(b) 90 km
(c) 30 km
(d) 250 km

## Subjective Problems

1. A man in a boat rowing away from a light house 100 m high takes 2 min to change the angle of elevation of the light house from $60^{\circ}$ to $45^{\circ}$. Find the speed of boat.
2. There are two windows in a house. A window of the house is at height of 2 m above the ground and the other window is 3 m vertically above the lower window. Ram and Shyam are sitting inside the two windows. At an instant, the angle of elevation of a balloon from these windows are observed as $45^{\circ}$ and $30^{\circ}$ respectively. Find the height of the balloon from the ground.
3. A tower in a city is 150 m high and a multistoreyed hotel at the city centre is 20 m high. The angle of elevation of the top of the tower at the top of the hotal is $5^{\circ}$. A builiding, $h$ metres high, is situated on the straight road connecting the tower with the city centre at a distance of 1.2 km from the tower. Find the value of $h$, if the top of the hotel, the top of the building and the top of the tower are in a straight line. Also, find the distance of the tower from the city centre.
(Use $\tan 5^{\circ}=0.0875 ; \tan 85^{\circ}=11.43$ )
4. An electrician has to repair an electric fault on a pole of height 5 m . He has to reach a point 1.3 m below the top of the pole to undertake
the repair work(see figure). What should be the length of the ladder that he should use which, when inclined at an angle of $60^{\circ}$ to the horizontal, would enable him to reach the required position? Also, how far from the foot of the pole he should place the foot of the ladder?
[Take $\sqrt{3}=1.73$ ].

5. A boy is standing on the ground and flying a kite with 75 m of string at an elevation of $45^{\circ}$. Another boy is standing on the roof of a 25 m high building and is flying his kite at an elevation of $30^{\circ}$. Both the boys are on opposite sides of the two kites. Find the length of the string that the second boy must have so that the two kites meet.
6. The angle of elevation $\theta$ of a vertical tower from a point on the ground is such that its tangent is $(5 / 12)$. On walking 192 metres towards the tower in the same straight line, the tangent of the angle of elevation $\phi$ is found to be (3/4).

Find the height of the tower.
7. The angles of depression of the top and the bottom of an 8 m tall building from the top of multistoreyed building are $30^{\circ}$ and $45^{\circ}$, respectively. Find the height of the multi-storeyed building and the distance between the two buildings.
8. Two stations due south of a leanings tower which leans towards the north are at distances $a$ and $b$ from its foot. If $\alpha, \beta$ be the elevations of the top of the tower from these stations, prove that its inclination $\theta$ to the horizontal is given by $\cot \theta=\frac{b \cot \alpha-a \cot \beta}{b-a}$.
9. From a window 15 m high above the ground in a street, the angles of elevation and depression of the top and foot of another house on the opposite side of the street are $30^{\circ}$ and $45^{\circ}$ respectively. Show that the height of the opposite house in 23.66 m . [Take, $\sqrt{3}=1.732$ ]
10. A carpenter makes stools for electricians with square top of side 0.5 m and at a height 1.5 m above the ground. Also, each leg is inclined at an angle of $60^{\circ}$ to the ground. Find the length of each leg and also the lengths of two steps in metres correct up to two places of decimals. [Take $\sqrt{3}=1.73$ ].

## CHAPTER

## Applications of Trigonometry

## Multiple Choice Questions

1. (a): Let $A B$ be the 7 m tall building and $C D$ be the $h$ metre high tower.
$\angle D B E=45^{\circ}$ and $\angle D A C=60^{\circ}$
In $\triangle D B E$, we have
$\tan 45^{\circ}=\frac{D E}{B E}$
$\Rightarrow 1=\frac{D E}{B E} \Rightarrow B E=D E$
$\Rightarrow B E=h-7$
In $\triangle A C D$, we have
$\tan 60^{\circ}=\frac{C D}{A C}$

$\Rightarrow \sqrt{3}=\frac{h}{A C}$
$\Rightarrow \sqrt{3} A C=h \Rightarrow \sqrt{3}(B E)=h \quad[\because A C=B E]$
$\Rightarrow \sqrt{3}(h-7)=h \Rightarrow \sqrt{3} h-7 \sqrt{3}=h$
$\Rightarrow \sqrt{3} h-h=7 \sqrt{3} \Rightarrow h=\frac{7 \sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$
$\Rightarrow h=\frac{7 \sqrt{3}(\sqrt{3}+1)}{2}=\frac{21+7 \sqrt{3}}{2}=\frac{21+7(1.732)}{2}$ $=\frac{21+12.124}{2}=16.562 \mathrm{~m}$
2. (b): Let height of pole be $h$ metre and length of shadow be also $h$ metre.
In $\triangle A B C$,
$\Rightarrow \tan \theta=\frac{A B}{B C}$
$\Rightarrow \tan \theta=\frac{A B}{A B} \quad[\because A B=B C]$

$\Rightarrow \tan \theta=1 \Rightarrow \tan \theta=\tan 45^{\circ} \Rightarrow \theta=45^{\circ}$
3. (a): Let height of tree $A B=h$ metre. It broke at $C$ its top $A$ touches the ground at $D$.
Now, $A C=C D$
and $A B=A C+B C=h$
$B D=10 \mathrm{~m}$
Now, in $\triangle B C D$ we have
$\tan 30^{\circ}=\frac{B C}{B D}$
$\Rightarrow \frac{1}{\sqrt{3}}=\frac{B C}{10} \Rightarrow B C=\frac{10}{\sqrt{3}}$

and $\cos 30^{\circ}=\frac{B D}{C D}$
$\Rightarrow \frac{\sqrt{3}}{2}=\frac{10}{C D} \Rightarrow C D=\frac{20}{\sqrt{3}}$
$\therefore \quad$ Height of tree $=B C+C D$

$$
=\frac{10}{\sqrt{3}}+\frac{20}{\sqrt{3}}=\frac{30}{\sqrt{3}}=10 \sqrt{3} \mathrm{~m}
$$

4. (a): Length of the string of the kite $A B=85 \mathrm{~m}$ $\tan \theta=\frac{15}{8}$
$\Rightarrow \cot \theta=\frac{8}{15}$
$\Rightarrow \operatorname{cosec}^{2} \theta-1=\frac{64}{225}$
$\Rightarrow \operatorname{cosec}^{2} \theta=1+\frac{64}{225}=\frac{289}{15}$
$\Rightarrow \operatorname{cosec} \theta=\sqrt{\frac{289}{15}}=\frac{17}{15} \Rightarrow \sin \theta=\frac{15}{17}$


In $\triangle A B C, \sin \theta=\frac{B C}{A B}$
$\Rightarrow \frac{15}{17}=\frac{B C}{85} \Rightarrow B C=75$
$\therefore \quad$ Height of kite $=75 \mathrm{~m}$
5. (b): In $\triangle A B C$, let $A B$ be the height of the tower.

Here $A C=100 \mathrm{~m}$
$\angle A C B=30^{\circ}$
In $\triangle A B C, \tan 30^{\circ}=\frac{A B}{A C}$
$\Rightarrow \frac{1}{\sqrt{3}}=\frac{A B}{100} \Rightarrow A B=\frac{100}{\sqrt{3}} \mathrm{~m}$

6. (b): Suppose height of the chimney is $h$ metres.

Let $A$ and $B$ be the point of observation and
$B C$ be $=x \mathrm{~m}$.
In $\triangle A C D$,
$\tan 30^{\circ}=\frac{C D}{A C}$
$\Rightarrow \frac{1}{\sqrt{3}}=\frac{h}{20+x}$

$\Rightarrow 20+x=h \sqrt{3}$
$\Rightarrow x=h \sqrt{3}-20$
Now, in $\triangle D B C, \tan 45^{\circ}=\frac{C D}{B C}$
$\Rightarrow 1=\frac{h}{x} \Rightarrow x=h$
From (1) and (2), we get
$h=h \sqrt{3}-20 \Rightarrow h \sqrt{3}-h=20$
$\therefore \quad h=\frac{20}{\sqrt{3}-1} \mathrm{~m}$
7. (d): Let the length of ladder $A B=h$ metres

Here $A C=9.5 \mathrm{~m}, \angle B A C=60^{\circ}$ In $\triangle A B C$,
$\cos 60^{\circ}=\frac{A C}{A B}$
$\Rightarrow \frac{1}{2}=\frac{9.5}{h}$

$\Rightarrow h=2 \times 9.5=19 \mathrm{~m}$
8. (a) : Let $A B$ be the rod and $B C$ be its shadow. In $\triangle A B C$,

$$
\begin{aligned}
& \tan \theta=\frac{A B}{B C} \\
\Rightarrow & \tan \theta=\frac{1}{\sqrt{3}}\left[\because \frac{A B}{B C}=\frac{1}{\sqrt{3}}\right] \\
\Rightarrow & \theta=30^{\circ}
\end{aligned}
$$


9. (b): Let $C D$ be the tower, $A$ and $B$ are points of observation on the ground. Let $B C$ be $x$.


In $\triangle A C D, \tan \alpha=\frac{C D}{A C}$

$$
\begin{equation*}
\Rightarrow \tan \alpha=\frac{h}{d+x} \Rightarrow d+x=\frac{h}{\tan \alpha} \tag{1}
\end{equation*}
$$

In $\triangle B C D, \tan \beta=\frac{C D}{B C}=\frac{h}{x}$
$\Rightarrow x=\frac{h}{\tan \beta}$
Subtracting (2) from (1), we get

$$
\begin{aligned}
& d+x-x=\frac{h}{\tan \alpha}-\frac{h}{\tan \beta} \\
\Rightarrow & d=h\left[\frac{1}{\tan \alpha}-\frac{1}{\tan \beta}\right]=h(\cot \alpha-\cot \beta) \\
\Rightarrow & h=\frac{d}{\cot \alpha-\cot \beta}
\end{aligned}
$$

10. (a): Here, $C D=20 \mathrm{~m}$ [Height of big pole]
$A B=14 \mathrm{~m}$ [Height of small pole]

$D E=C D-C E \Rightarrow D E=C D-A B[\because A B=C E]$
$\Rightarrow D E=20-14=6 \mathrm{~m}$
In $\triangle B D E, \sin 30^{\circ}=\frac{D E}{B D}$
$\Rightarrow \frac{1}{2}=\frac{6}{B D} \Rightarrow B D=12 \mathrm{~m}$
$\therefore$ Length of wire $=12 \mathrm{~m}$
11. (a) : Let $C$ and $D$ be the position of the men and $A B$ be the height of flagstaff.


In $\triangle A B C, \tan 30^{\circ}=\frac{A B}{B C}$
$\Rightarrow \frac{1}{\sqrt{3}}=\frac{20}{B C} \Rightarrow B C=20 \sqrt{3}$
In $\triangle A B D, \tan 60^{\circ}=\frac{A B}{B D}$
$\Rightarrow \sqrt{3}=\frac{20}{B D} \Rightarrow B D=\frac{20}{\sqrt{3}}$
Distance between the men, $C D=B C+B D$
$=20 \sqrt{3}+\frac{20}{\sqrt{3}}=\frac{60+20}{\sqrt{3}}=\frac{80}{\sqrt{3}}$
$=\frac{80 \sqrt{3}}{3}=\frac{80 \times 1.73}{3}=46.19 \mathrm{~m}$
12. (b): Let $A B$ be the tower. Let $C$ and $D$ be two points at distance $a$ and $b$ respectively from the base of the tower. In $\triangle A B C$,

$$
\begin{equation*}
\tan \theta=\frac{A B}{A C} \Rightarrow \tan \theta=\frac{h}{a} \tag{1}
\end{equation*}
$$



In $\triangle A B D$,
$\tan \left(90^{\circ}-\theta\right)=\frac{A B}{A D} \Rightarrow \cot \theta=\frac{h}{b}$
Multiplying (1) and (2), we have
$\tan \theta \times \cot \theta=\frac{h}{a} \times \frac{h}{b} \Rightarrow 1=\frac{h^{2}}{a b} \Rightarrow h=\sqrt{a b}$
13. (c) : Let $C$ and $D$ be the earth stations and $A B$ be the height of the satellite


In $\triangle A B C, \tan 30^{\circ}=\frac{A B}{B C}$
$\Rightarrow \frac{1}{\sqrt{3}}=\frac{h}{4000+x} \Rightarrow 4000+x=\sqrt{3} h$
In $\triangle A B D, \tan 60^{\circ}=\frac{A B}{B D}$
$\Rightarrow \sqrt{3}=\frac{h}{x} \Rightarrow x=\frac{h}{\sqrt{3}}$
From (1) and (2), we get
$4000+\frac{h}{\sqrt{3}}=\sqrt{3} h \Rightarrow 4000 \sqrt{3}=3 h-h$
$\Rightarrow 2 h=4000 \sqrt{3} \Rightarrow h=2000 \sqrt{3}$
$\Rightarrow h=2000(1.732)=3464 \mathrm{~km}$
14. (c) : In $\triangle A D M, \sin 30^{\circ}=\frac{A M}{A D}$
$\Rightarrow \frac{1}{2}=\frac{A M}{12} \Rightarrow A M=6 \mathrm{~cm}$


Area of parallelogram $A B C D=C D \times A M$
$\therefore C D \times A M=60 \quad\left[\because\right.$ Area of $\left.\| \mathrm{gm}=60 \mathrm{~cm}^{2}\right]$
$\Rightarrow C D \times 6=60 \Rightarrow C D=10 \mathrm{~cm}$
15. (a): Let $A B$ be height of tree and $B C$ be its shadow. Again, let $P Q$ be height of pole and $Q R$ be its shadow.


At the same time the angle of elevation of tree and pole is equal
i.e., $\triangle A B C \sim \triangle P Q R$
$\therefore \frac{A B}{B C}=\frac{P Q}{Q R} \Rightarrow \frac{6}{4}=\frac{P Q}{50}$
$\Rightarrow P Q=\frac{50 \times 6}{4} \Rightarrow P Q=75 \mathrm{~m}$

16. (d):


In $\triangle C D E$,
$\tan \left(90^{\circ}-\theta\right)=\frac{h}{a / 2}$
$\Rightarrow \cot \theta=\frac{2 h}{a}$
In $\triangle A B E, \tan \theta=\frac{A B}{E A}=\frac{2 h}{a / 2}$
$\Rightarrow \tan \theta=\frac{4 h}{a}$
Multiplying (1) by (2), we get
$\cot \theta \times \tan \theta=\frac{2 h}{a} \times \frac{4 h}{a}$
$\Rightarrow 1=\frac{8 h^{2}}{a^{2}} \Rightarrow h=\frac{a}{2 \sqrt{2}}$
17. (d): Let $A B$ the height of the cloud above the lake be $H$ metre.

$A B=B Q=H \mathrm{~m}, C D=P B=200 \mathrm{~m}$
$A P=A B-P B=H-200$
$P Q=P B+B Q=200+H$

In $\triangle A P D, \tan 30^{\circ}=\frac{A P}{P D}$
$\Rightarrow \frac{1}{\sqrt{3}}=\frac{H-200}{P D} \Rightarrow P D=(H-200) \sqrt{3}$
In $\triangle P D Q, \tan 60^{\circ}=\frac{P Q}{P D}$
$\Rightarrow \sqrt{3}=\frac{H+200}{P D} \Rightarrow P D=\frac{H+200}{\sqrt{3}}$
From (1) and (2), we get

$$
\begin{aligned}
& (H-200) \sqrt{3}=\frac{H+200}{\sqrt{3}} \\
\Rightarrow & 3(H-200)=H+200 \\
\Rightarrow & 3 H-600=H+200 \\
\Rightarrow & 2 H=800 \Rightarrow H=400 \mathrm{~m}
\end{aligned}
$$

18. (c) : Let $H$ metre be the height of the cloud above water level.


Here, $A B=h, A C=B Q=x$
In $\triangle A C P, \tan \theta=\frac{P C}{A C}=\frac{H-h}{x}$
$\Rightarrow x=\frac{H-h}{\tan \theta}$
In $\triangle A C R, \tan 45^{\circ}=\frac{C R}{A C} \Rightarrow \tan 45^{\circ}=\frac{H+h}{x}$
$\Rightarrow x=\frac{H+h}{\tan 45^{\circ}}$
From (1) and (2), we have

$$
\frac{H+h}{\tan 45^{\circ}}=\frac{H-h}{\tan \theta} \Rightarrow \frac{H+h}{H-h}=\frac{\tan 45^{\circ}}{\tan \theta}
$$

Using componedo and dividendo, we get

$$
\begin{aligned}
& \frac{H+h+H-h}{H+h-H+h}=\frac{\tan 45^{\circ}+\tan \theta}{\tan 45^{\circ}-\tan \theta} \\
& \Rightarrow \quad \frac{2 H}{2 h}=\frac{1+\tan \theta}{1-\tan \theta} \Rightarrow H=h\left(\frac{1+\tan \theta}{1-\tan \theta}\right) \\
& \therefore \quad H=h \tan \left(45^{\circ}+\theta\right) \\
& \quad\left[\because \frac{1+\tan \theta}{1-\tan \theta}=\tan \left(45^{\circ}+\theta\right)\right]
\end{aligned}
$$

19. (b): Let $A B$ be the height of the rock and $C D$ be the height of tower.

$C D=B E=100 \mathrm{~m}$,
$A B=H$ metre
$\therefore \quad A E=A B-B E=H-100, C E=B D=x$
In $\triangle A C E, \tan 30^{\circ}=\frac{A E}{C E}$
$\Rightarrow \frac{1}{\sqrt{3}}=\frac{H-100}{x} \Rightarrow x=\sqrt{3}(H-100)$
In $\triangle A B D, \tan 45^{\circ}=\frac{A B}{B D}$
$\Rightarrow 1=\frac{H}{x} \Rightarrow x=H$
From (1) and (2), we have
$H=\sqrt{3}(H-100) \Rightarrow H=\sqrt{3} H-100 \sqrt{3}$
$\Rightarrow \sqrt{3} H-H=100 \sqrt{3} \Rightarrow H=\frac{100 \sqrt{3}}{\sqrt{3}-1}$
$\Rightarrow H=\frac{100 \sqrt{3}(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}=\frac{100 \sqrt{3}(\sqrt{3}+1)}{2}$
$\Rightarrow H=50 \sqrt{3}(\sqrt{3}+1) \Rightarrow H=50(3+\sqrt{3}) \mathrm{m}$
20. (d): Let $O$ be the starting point of Mr. Anna Hazzare Padyatra party.

$O A=150 \mathrm{~km}, A C=120 \mathrm{~km}$
In $\triangle A B C, \sin 45^{\circ}=\frac{B C}{A C}$
$\frac{1}{\sqrt{2}}=\frac{B C}{120} \Rightarrow B C=60 \sqrt{2}$
and $\cos 45^{\circ}=\frac{A B}{A C} \Rightarrow \frac{1}{\sqrt{2}}=\frac{A B}{120} \Rightarrow A B=60 \sqrt{2}$
$\therefore \quad O B=O A+A B=150+60 \sqrt{2}$
$\Rightarrow O B=150+60(1.414)=234.84$
and $B C=60 \sqrt{2}=60(1.414)=84.84$

In $\triangle O B C, O C^{2}=O B^{2}+B C^{2}$
$\Rightarrow O C^{2}=(234.84)^{2}+(84.84)^{2}$
$\Rightarrow O C^{2}=55149.82+7197.82$
$\Rightarrow O C^{2}=62347.64$
$\Rightarrow \mathrm{OC}=\sqrt{62347.64}=249.69$
$\therefore \quad O C=250 \mathrm{~km}$ (approx)
Thus, the distance between starting point to the final point is 250 km approx.

## Subjective Problems

1. Let us assume that the light house $A B$ be 100 m and $C, D$ be the positions of the man when angle of elevation changes from $60^{\circ}$ to $45^{\circ}$. The man has covered a distance $C D$ in 2 min .

$$
\text { Speed }=\frac{\text { Distance }}{\text { Time }}
$$

$\therefore \quad$ Speed $=\frac{C D}{2}$
In $\triangle A B C$, we have

$$
\begin{aligned}
& \tan 60^{\circ}=\frac{A B}{B C} \\
\Rightarrow & \sqrt{3}=\frac{100}{B C} \\
\Rightarrow & B C=\frac{100}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \\
\Rightarrow & B C=\frac{100 \sqrt{3}}{3}
\end{aligned}
$$



In $\triangle A B D$, we have

$$
\begin{aligned}
\tan 45^{\circ} & =\frac{A B}{B D} \\
\Rightarrow 1 & =\frac{100}{B D} \\
\Rightarrow \quad B D & =100 \mathrm{~m} \\
\text { As } C D & =D B-B C \\
& =100-\frac{100 \sqrt{3}}{3} \\
& =100\left(1-\frac{\sqrt{3}}{3}\right)=100\left(\frac{3-\sqrt{3}}{3}\right) \\
\therefore \quad \text { Speed } & =\frac{C D}{2}=\frac{100\left(\frac{3-\sqrt{3}}{3}\right)}{2} \\
& =50\left(\frac{3-\sqrt{3}}{3}\right)^{2}
\end{aligned}
$$

Hence, the required speed of boat is $\frac{50}{3}(3-\sqrt{3}) \mathrm{m} / \mathrm{min}$
2. Let $H$ be the height of the balloon from the ground and $C$ and $D$ be the position of the windows.


At $C$ and $D$, angle of elevation are $\angle E C G=45^{\circ}$ and $\angle F D G=30^{\circ}$
Let $C E=D F=x \mathrm{~m}$ and $F G=h \mathrm{~m}$
In $\triangle C E G$, we have

$$
\begin{align*}
& \tan 45^{\circ}=\frac{E G}{E C} \\
\Rightarrow & 1=\frac{E F+F G}{E C} \\
\Rightarrow & 1=\frac{3+h}{x} \\
\Rightarrow & x=3+h \tag{i}
\end{align*}
$$

In $\triangle D F G$, we have

$$
\begin{align*}
& \tan 30^{\circ}=\frac{G F}{D F} \\
& \Rightarrow \frac{1}{\sqrt{3}}=\frac{h}{x} \\
& \Rightarrow x=\sqrt{3} h \tag{ii}
\end{align*}
$$

substituting $x=\sqrt{3} h$ in (i), we get

$$
\begin{aligned}
& \sqrt{3} h=3+h \\
\Rightarrow & \sqrt{3} h-h=3 \Rightarrow h(\sqrt{3}-1)=3 \Rightarrow h=\frac{3}{\sqrt{3}-1} \\
\Rightarrow & h=\frac{3}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} \\
\Rightarrow & h=\frac{3(\sqrt{3}+1)}{3-1} \Rightarrow h=\frac{3 \times(1.732+1)}{2} \\
\Rightarrow & h=\frac{3 \times 2.732}{2} \Rightarrow h=4.098 \mathrm{~m}
\end{aligned}
$$

Hence, the height of the balloon from the ground is

$$
\begin{aligned}
H & =E A+F E+h \\
\Rightarrow \quad H & =2+3+4.098 \Rightarrow H=9.098 \mathrm{~m}
\end{aligned}
$$

3. Let the tower be $A B$, hotel be $E F$ and building be $C D$.

$A B=150 \mathrm{~m}, E F=20 \mathrm{~m}, C D=h, B D=1200 \mathrm{~m}$
Let $D F=x \mathrm{~m}$
$P E=B F=B D+D F=(1200+x) \mathrm{m}$
In right $\triangle A P E$,

$$
\begin{aligned}
& \tan \angle A E P=\frac{A P}{P E} \\
\Rightarrow & \frac{A P}{P E}=\tan 5^{\circ} \Rightarrow \frac{A B-P B}{B F}=\tan 5^{\circ} \\
\Rightarrow & \frac{150-20}{1200+x}=\tan 5^{\circ} \Rightarrow \frac{130}{1200+x}=0.0875 \\
\Rightarrow & 130=(1200+x)(0.0875) \\
\Rightarrow & 130=105+0.0875 x \\
\Rightarrow & 0.0875 x=130-105=25 \Rightarrow x=\frac{25}{0.0875} \\
\Rightarrow & x=285.7=286 \mathrm{~m} \text { nearly }
\end{aligned}
$$

Now, in right $\triangle C Q E$,
$C Q=C D-D Q=h-E F=h-20$

$$
\begin{aligned}
& \text { Also, } \frac{C Q}{Q E}=\tan 5^{\circ} \Rightarrow C Q=Q E \tan 5^{\circ} \\
& \Rightarrow \quad h-20=286 \times 0.0875 \Rightarrow h-20=25 \\
& \Rightarrow \quad h=20+25 \Rightarrow h=45 \mathrm{~m} \\
& \therefore \quad \text { Distance of the tower }=B D+D F
\end{aligned}
$$

$$
=B D+Q E \quad \ldots(\because D F=Q E)
$$

$$
=1200+286=1486 \mathrm{~m} \text { nearly }
$$

4. In figure, the electrician is required to reach the point $B$ on the pole $A D$.
So, $B D=A D-A B=(5-1.3)=3.7 \mathrm{~m}$
Here, $B C$ represents the ladder. We need to find its length, i.e., the hypotenuse of the right triangle $B D C$.
In right $\triangle B D C$, we have

$$
\sin 60^{\circ}=\frac{B D}{B C} \Rightarrow \frac{3.7}{B C}=\frac{\sqrt{3}}{2}
$$

$\therefore \quad B C=\frac{3.7 \times 2}{\sqrt{3}}=4.28 \mathrm{~m}$ (Approx.)
i.e., the length of the ladder should be 4.28 m

Now, $\frac{D C}{B D}=\cot 60^{\circ}=\frac{1}{\sqrt{3}}$
i.e., $D C=\frac{3.7}{\sqrt{3}}=2.14 \mathrm{~m}$ (Approx)


Therefore, he should place the foot of the ladder at a distance of 2.14 m from the pole.
5. Let the first boy be at $A$ and second boy be at $B$. $A$ is on the ground and $B$ is 25 m above the ground.


Let $K$ be the point where the two kites meet. Draw $K M \perp A N$ and $B L \perp K M$.
$\angle K A M=45^{\circ}, A K=75 \mathrm{~m}, \angle K B L=30^{\circ}$
$\therefore \quad L M=B N=25 \mathrm{~m}$
Now, in $\triangle K A M, \frac{K M}{A K}=\sin 45^{\circ}=\frac{1}{\sqrt{2}}$
$\Rightarrow K M=\frac{A K}{\sqrt{2}}=\frac{75}{\sqrt{2}}=\frac{75 \sqrt{2}}{2} \mathrm{~m}$
$\therefore K L=K M-L M=\left(\frac{75 \sqrt{2}}{2}-25\right) \mathrm{m}$
In $\triangle K L B, \frac{K L}{K B}=\sin 30^{\circ}$
$\Rightarrow\left(\frac{\frac{75 \sqrt{2}}{2}-25}{K B}\right)=\frac{1}{2} \quad[U \operatorname{sing}(1)]$
$\Rightarrow K B=2\left(\frac{75 \sqrt{2}}{2}-25\right)=(75 \sqrt{2}-50) \mathrm{m}$
$\Rightarrow K B=75 \times 1.414-50=106.05-50=56.05 m$
Hence, the required length of string is 56.05 m .
6. Let $P Q$ be the tower and let $A$ and $B$ be the given points of observation on the ground.
Then, $\angle P A Q=\theta, \angle P B Q=\phi$,
$\angle A P Q=90^{\circ}, A B=192 \mathrm{~m}$,
$\tan \theta=\frac{5}{12}$ and $\tan \phi=\frac{3}{4}$

Let $P Q=h$ metres and $B P=x$ metres
From right $\triangle A P Q$, we have
$\tan \theta=\frac{P Q}{A P}$
$\Rightarrow \frac{h}{192+x}=\frac{5}{12}$
$\Rightarrow 5 x=(12 h-960)$
$\Rightarrow x=\frac{12 h-960}{5}$
From right $\triangle B P Q$, we have
$\tan \phi=\frac{P Q}{B P} \Rightarrow \frac{h}{x}=\frac{3}{4}$
$\Rightarrow 3 x=4 h \Rightarrow x=\frac{4 h}{3}$
From (1) and (2), we get
$\frac{12 h-960}{5}=\frac{4 h}{3} \Rightarrow 36 h-2880=20 h$
$\Rightarrow 16 h=2880 \Rightarrow h=180 \mathrm{~m}$
Hence, the height of the tower $=180 \mathrm{~m}$
7. In figure, $P C$ denotes the multistoreyed building and $A B$ denotes the 8 m tall building.


In right $\triangle P B D$, we have
$\frac{P D}{B D}=\tan 30^{\circ} \Rightarrow \frac{P D}{B D}=\frac{1}{\sqrt{3}} \Rightarrow B D=P D \sqrt{3}$
In right $\triangle P A C$, we have

$$
\frac{P C}{A C}=\tan 45^{\circ} \Rightarrow \frac{P C}{A C}=1 \Rightarrow P C=A C
$$

Also, $P C=P D+D C$, therefore, $P D+D C=A C$
Since, $A C=B D$ and $D C=A B=8 \mathrm{~m}$, we get

$$
P D+8=B D=P D \sqrt{3}
$$

This gives, $P D=\frac{8}{\sqrt{3}-1}$
$=\frac{8(\sqrt{3}+1)}{(\sqrt{3}+1)(\sqrt{3}-1)}=4(\sqrt{3}+1)$
So, the height of the multi-storeyed building is $P C=P D+D C=8+\{4(\sqrt{3}+1)\}=4(3+\sqrt{3}) \mathrm{m}$ and distance between the two building is also $4(3+\sqrt{3}) \mathrm{m} .(\because P C=A C)$
8. Let $A B$ be the leaning tower and let $C$ and $D$ be two given stations at distances $a$ and $b$ respectively from the foot $A$ of the tower.


Let $A E=x$ and $B E=h$
In $\triangle A E B$, we have

$$
\begin{align*}
& \quad \tan \theta=\frac{B E}{A E} \Rightarrow \tan \theta=\frac{h}{x} \\
& \Rightarrow x=h \cot \theta  \tag{1}\\
& \text { In } \triangle C E B, \text { we have }
\end{align*}
$$

$\tan \alpha=\frac{B E}{C E} \Rightarrow \tan \alpha=\frac{h}{a+x}$
$\Rightarrow a+x=h \cot \alpha$
$\Rightarrow x=h \cot \alpha-a$
In $\triangle D E B$, we have

$$
\begin{align*}
& \tan \beta=\frac{B E}{D E} \Rightarrow \tan \beta=\frac{h}{b+x}  \tag{2}\\
\Rightarrow & b+x=h \cot \beta \\
\Rightarrow & x=h \cot \beta-b \tag{3}
\end{align*}
$$

On equating the values of $x$ obtained from equations (1) and (2), we have
$h \cot \theta=h \cot \alpha-a$
$\Rightarrow h(\cot \alpha-\cot \theta)=a$
$\Rightarrow \quad h=\frac{a}{\cot \alpha-\cot \theta}$
On equating the values of $x$ obtained from equations (1) and (3), we get
$h \cot \theta=h \cot \beta-b$
$\Rightarrow h(\cot \beta-\cot \theta)=b$
$\Rightarrow h=\frac{b}{\cot \beta-\cot \theta}$
On equating the values of $h$ from equations (4) and (5), we get

$$
\begin{aligned}
& \frac{a}{\cot \alpha-\cot \theta}=\frac{b}{\cot \beta-\cot \theta} \\
\Rightarrow & a(\cot \beta-\cot \theta)=b(\cot \alpha-\cot \theta) \\
\Rightarrow & (b-a) \cot \theta=b \cot \alpha-a \cot \beta \\
\Rightarrow & \cot \theta=\frac{b \cot \alpha-a \cot \beta}{b-a}
\end{aligned}
$$

9. Let us assume that the window be at point $A$ at height of 15 m above the ground and $C D$ be the house on the opposite side of the street, such that the angle of elevation of the top $D$ of the
house $C D$ is $30^{\circ}$ and the angle of depression of the foot $C$ is $45^{\circ}$.

$\therefore \quad \angle D A E=30^{\circ}$
and $\angle C A E=\angle A C B=45^{\circ}$ (Alternate angles)
Let height of the house $C D$ be $h \mathrm{~m}$.
Clearly, $E D=C D-C E$

$$
=C D-A B=h-15(\because C E=A B)
$$

In $\triangle A E C$, we have

$$
\begin{aligned}
& \tan 45^{\circ}=\frac{E C}{A E} \\
\Rightarrow & 1=\frac{15}{A E} \Rightarrow A E=15 \mathrm{~m}
\end{aligned}
$$

Now, in $\triangle A E D$, we have

$$
\begin{aligned}
& \tan 30^{\circ}=\frac{D E}{A E} \\
\Rightarrow & \frac{1}{\sqrt{3}}=\frac{h-15}{15} \Rightarrow 15=\sqrt{3}(h-15) \\
\Rightarrow & 15=h \sqrt{3}-15 \sqrt{3} \Rightarrow h \sqrt{3}=15(1+\sqrt{3}) \\
\Rightarrow & h=\frac{15(\sqrt{3}+1)}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \\
& =\frac{15 \sqrt{3}(\sqrt{3}+1)}{3}=5 \sqrt{3}(\sqrt{3}+1)=15+5 \sqrt{3} \\
& =15+5 \times 1.732=15+8.660 \\
\therefore \quad & h=23.66 \mathrm{~m}
\end{aligned}
$$

Hence, the height of the opposite house is 23.66 m.
10. As shown in figure, $A X$ and $B Y$ are the two legs at an angle of $60^{\circ}$ to the ground. $C D$ and $E F$ are two steps. $A R$ and $B S$ are perpendiculars on the base $X Y$.

$A R$ meets $C D$ at $L$ and $E F$ at $P$.
$B S$ meets $C D$ at $M$ and $E F$ at $Q$.
We have, $A R=B S=1.5 \mathrm{~m}, A B=0.5 \mathrm{~m}$
$A L=L P=P R=B M=M Q=Q S=0.5 \mathrm{~m}$
From right angled $\triangle A X R$,
$\sin 60^{\circ}=\frac{A R}{A X} \Rightarrow \frac{\sqrt{3}}{2}=\frac{1.5}{A X}$
$\Rightarrow A X=\frac{1.5 \times 2}{\sqrt{3}}=\frac{3}{\sqrt{3}} \Rightarrow A X=\sqrt{3} \mathrm{~m}$
$\Rightarrow A X=1.73 \mathrm{~m} \Rightarrow A X=B Y=1.73 \mathrm{~m}$
In right angled $\triangle A C L$ and $\triangle A E P, \angle A C L=60^{\circ}$ and $\angle A E P=60^{\circ}$
Then $\frac{A L}{C L}=\tan 60^{\circ}$ and $\frac{A P}{E P}=\tan 60^{\circ}$
$\Rightarrow \frac{0.5}{C L}=\sqrt{3}$ and $\frac{1}{E P}=\sqrt{3}$
$\Rightarrow C L=\frac{0.5}{\sqrt{3}}$ and $E P=\frac{1}{\sqrt{3}}$
$\Rightarrow C L=\frac{0.5 \times \sqrt{3}}{3}$ and $E P=\frac{\sqrt{3}}{3}$
Similarly, $M D=\frac{0.5 \times \sqrt{3}}{3}$ and $Q F=\frac{\sqrt{3}}{3}$
Now, $C D=C L+L M+M D$
$=\frac{0.5 \times \sqrt{3}}{3}+0.5+\frac{0.5 \times \sqrt{3}}{3}$
$=0.5+\frac{\sqrt{3}}{3}=0.5+\frac{1.73}{3}$
$=0.5+0.577($ approx $)=1.077 \mathrm{~m}$
Again, $E F=E P+P Q+Q F=\frac{\sqrt{3}}{3}+0.5+\frac{\sqrt{3}}{3}$
$=0.5+\frac{2 \sqrt{3}}{3}=0.5+\frac{2 \times 1.73}{3}$
$=0.5+\frac{3.46}{3}=0.5+1.153=1.653 \mathrm{~m}$
Hence, length of each leg $=1.73 \mathrm{~m}$
and length of step $C D=1.077 \mathrm{~m}$
and length of step $E F=1.653 \mathrm{~m}$

